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A Simplified Computer Solution for the Flexibility Matrix of Contacting Teeth for Spiral Bevel Gears

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TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

In order to calculate the dynamic loads between contacting teeth of spiral bevel gears, it is necessary to compute first the elastic deformation of the tooth contacting surface under an unit normal load located at any arbitrary point on the surface. This was accomplished by Chao(1) using the finite element method. To insure accuracy, the finite element method was employed not only for the gear tooth but also for the gear wheel, hub and shaft. Such an approach necessitates the use of a large number of elements, and the computer cost was found to be prohibitive.

A simplified method is developed in this report to obtain the flexibility matrix, i.e., the deflection along the direction normal to the surface at each grid point on the tooth face due to an unit normal force acting on this point. In this analysis, an existing finite element code SAPIV was used to compute the deformation of the tooth itself. For the deformation of the gear wheel and shaft, conventional beam theory is employed. The total deformation is obtained by using the principle of superposition.

CHAPTER II

ELASTIC COEFFICIENTS

In order to obtain the deflection in the normal direction due to an unit normal force acting on a given grid point(Fig. 2-1), one can separate the gear into two parts: (1) tooth (2) wheel and shaft(Fig. 2-2). After obtaining the deflections from the above two parts, one can superimpose them to obtain the total deflection. Then the deflection in the normal direction at each grid point on the tooth face can be determined.

2.1 Deflection of Tooth Fixed on Root Cone

By fixing the root of the tooth, the finite element program SAPIV can be used to calculate the displacements of each grid point on the tooth surface for an unit normal load applied at any arbitrary grid point. In this calculation, three dimensional solid elements(eight node brick) are used. Fig. 2-3 shows the node number of a typical spiral bevel tooth which has a profile geometry indentical to that used by Chao(1). Appendix A gives two samples of input data for SAPIV.

2.2 Deflection of Wheel and Shaft

Let us consider the wheel and shaft as a beam. There are two axial planes perpendicular to each other and a transverse plane for the pinion or gear shaft as shown in Fig. 2-4(p. 25) for calculating the displacements in the xy plane. The boundary conditions are (see Fig. 2-5):

$$(D_y)_C = (D_y)_D = 0$$

$$(D_x)_C = 0 \qquad (2.1)$$

$$(\theta_x)_E = 0$$

where $(D_y)_C$ is the displacement of point C along the y axis and $(\theta_x)_E$ is the angular displacement about the x axis at point E.

The unit normal force acting at any grid point G (Fig. 2-6) can be replaced by a set of forces and couples applied at the center of the cross section cut by the transverse plane (Fig. 2-4) through the grid point. The principle of superposition is applied here, and the deflection due to the force or the couple can be computed separately as shown in the following sections. These deflections can then be added algebraically to obtain the total deflection.

2.2.1 Deflections Due To Bending Moment

Fig. 2-7 is the free body diagrm the unit normal force acting at a grid point on the gear contacting surface has been replaced by a set of forces and couples applied on the center of the cross section cut by the transverse plane through the grid point at a distance L from the y axis.

To obtain the deflection and slope at L, the beam is divided into 3 portions.

(1) $b \leq x < d$

Considering the force equilibrium of the portion of the beam between B and E (Fig. 2-8), one obtains

$$M_2 = P\,(b - L) - M_0 \tag{2.2}$$

The moment of inertia of the cross sectional area with respect to the neutral axis is

$$I_1 = \frac{\pi(r_s^4 - r_i^4)}{4} \tag{2.3}$$

To obtain the deflection and slope at B due to the force P and moment M_2, one can consider the deflection owing to P or M_2 separately. Referring to beam deflection table (2) and using the principle of superposition the

following results are obtained

$$y_B = \frac{b - c}{6EI_1}[2P(c - b)(d - b) + M_2(2d - 3b + c)] \qquad (2.4)$$

$$y'_B = \frac{1}{6EI_1}[P(c - b)(2d - 3b + c) + 2M_2(d + 2c - 3b)] \qquad (2.5)$$

where E is the modulus of elasticity.

Appendix B shows a detailed derivation of the deflection and slope at point B by solving differential equation of the elastic line B C D E.

(2) $a < L < x < b$

Fig 2-9 is the free body diagram for the forces acting on the portion of the beam to the right of point L. One obtains

$$M_3 = P(x - L) - M_0 \qquad (2.6)$$

Radius of pitch cone on the cross section passed by the transverse plane at a distance x from the y axis is

$$r_p = x\tan\theta_p \qquad (2.7)$$

where θ_p is the pitch angle.

The moment of inertia of the cross-sectional area with respect to the neutral axis at a distance x from the y axis is

$$I_3 = \frac{\pi(x^4 \tan^4\theta_p - r_i^4)}{4} \tag{2.8}$$

The differential equation is

$$y'' = -\frac{M_3}{EI_3} \tag{2.9}$$

$$= \frac{-4P}{\pi E \tan^4\theta_p} \left[\frac{x - (L + \frac{M_0}{P})}{x^4 - \frac{r_i^4}{\tan^4\theta_p}}\right]$$

Let

$$A_5 = \frac{-4P}{\pi E \tan^4\theta_p} \tag{2.10}$$

$$A_6 = L + \frac{M_0}{P} \tag{2.11}$$

$$A_7 = \frac{r_i}{\tan\theta_p} \tag{2.12}$$

If $r_i \neq 0$

$$y' = A_5 \int \left(\frac{x - A_6}{x^4 - A_7^4}\right) dx + C_5 \tag{2.13}$$

$$y' = A_5 \int \left(\frac{D_1}{x - A_7} + \frac{D_2}{x + A_7} + \frac{D_3 x + D_4}{x^2 + A_7^2}\right) dx + C_5 \tag{2.14}$$

$$= A_5 \left[D_1 \ln|x - A_7| + D_2 \ln|x + A_7| + \frac{D_3}{2} \ln(x^2 + A_7^2) + \frac{D_4}{A_7} \tan^{-1} \frac{x}{A_7}\right] + C_5$$

where

$$D_1 = \frac{1}{4A_7^3}(A_7 - A_6) \tag{2.15}$$

$$D_2 = \frac{1}{4A_7^3}(A_6 + A_7) \tag{2.16}$$

$$D_3 = -\frac{1}{2A_7^2} \tag{2.17}$$

$$D_4 = \frac{A_6}{2A_7^2} \tag{2.18}$$

C_5 can be determined from the condition

$$y' = y'_B \quad \text{at } x = b$$

One obtains

$$C_5 = y'_B - A_5[D_1 \ln|b - A_7| + D_2 \ln|b + A_7| + \frac{D_3}{2}\ln(b^2 + A_7^2) + \frac{D_4}{A_7}\tan^{-1}\frac{b}{A_7}] \quad (2.19)$$

Separating variables and integrating one obtains

$$y = A_5[D_1(x - A_7)\ln|x - A_7| - D_1(x - A_7) + D_2(x + A_7)\ln|x + A_7| - D_2(x + A_7) + \frac{D_3 x}{2}\ln(x^2 + A_7^2) - D_3 x + D_3 A_7 \tan^{-1}\frac{x}{A_7} + \frac{D_4 x}{A_7}\tan^{-1}\frac{x}{A_7} - \frac{D_4}{2}\ln[(\frac{x}{A_7})^2 + 1]] + C_5 x + C_6 \quad (2.20)$$

At $x = b$, $y = y_B$, one obtains

$$C_6 = y_B - A_5[D_1(b - A_7)[\ln|b - A_7| - 1] + D_2(b + A_7)[\ln|b + A_7| - 1] + \frac{D_3 b}{2}[\ln(b^2 + A_7^2) - 2] + \tan^{-1}\frac{b}{A_7}(D_3 A_7 + \frac{D_4 b}{A_7}) - \frac{D_4}{2}\ln[(\frac{b}{A_7})^2 + 1]] - C_5 b \quad (2.21)$$

Setting x = L, one obtains the deflection and slope of point L

$$y_L = A_5\{D_1(L - A_7)[\ln|L - A_7| - 1] + D_2(L + A_7)[\ln|L + A_7| - 1] + \frac{D_3 L}{2}[\ln(L^2 + A_7^2) - 2] + \tan^{-1}\frac{L}{A_7}(D_3 A_7 + \frac{D_4 L}{A_7}) - \frac{D_4}{2}\ln[(\frac{L}{A_7})^2 + 1]\} + C_5 L + C_6 \tag{2.22}$$

$$y'_L = A_5[D_1 \ln|L - A_7| + D_2 \ln|L + A_7| + \frac{D_3}{2}\ln(L^2 + A_7^2) + \frac{D_4}{A_7}\tan^{-1}\frac{L}{A_7}] + C_5 \tag{2.23}$$

If $r_i = 0$

$$y' = A_5\int(\frac{x - A_6}{x^4})dx + C'_5$$

$$= -\frac{A_5}{6x^3}(3x - 2A_6) + C'_5 \tag{2.24}$$

$$y = \frac{A_5}{6x^2}(3x - A_6) + C'_5 x + C'_6 \tag{2.25}$$

By knowing $y' = y'_B$ and $y = y_B$ at $x = b$, one obtains

$$C'_5 = y'_B + \frac{A_5}{6b^3}(3b - 2A_6) \tag{2.26}$$

$$C'_6 = y_B - \frac{A_5}{6b^2}(3b - A_6) - C'_5 b \tag{2.27}$$

Replacing x by L into equations (2.24) and (2.25)

$$y_L = \frac{A_5}{6L^2}(3L - A_6) + C'_5 L + C'_6 \tag{2.28}$$

$$y'_L = -\frac{A_5}{6L^3}(3L - 2A_6) + C'_5 \tag{2.29}$$

(3) $a < x < L$

Setting $x = a$ into equations (2.14) and (2.20)(or (2.24) and (2.25) as $r_i = 0$), one obtains y_A and y'_A. Then

$$y_L = y_A + (L - a)y'_A \tag{2.30}$$

$$y'_L = y'_A \tag{2.31}$$

The displacement of grid point G can be calculated from the displacement of L by virtue of the fact that

G and L lie in the same transverse plane which is assumed to remain plane after deformation.

Fig. 2-10 shows how the grid point G moves to the final position following the displacement of point L. Here

$$D = |y_L| \tag{2.32}$$

$$\theta = |y_L'| \tag{2.33}$$

Hence, one can obtain the displacement of the grid point G by rotation about point L and translation.

2.2.2 Deflections Due to Shearing Strain

Referring to Fig. 2-7 again, one can divide the beam into 3 portions for derivation.

(1) $b < x < c$

$$I_1 = \frac{\pi(r_s^4 - r_i^4)}{4}$$

The maximum shear stress τ_{max} is induced on neutral plane. So

$$b_1 = 2(r_s - r_i) \tag{2.34}$$

$$\tau_{max} = \frac{P}{I_1 b_1} \int_{r_i}^{r_s} y dA$$

$$= \frac{8(r_s^3 - r_i^3)P}{3\pi b_1(r_s^4 - r_i^4)} \tag{2.35}$$

Then the slope of the elastic line becomes

$$\frac{dy}{dx} = \frac{\tau_{max}}{G} \tag{2.36}$$

where G is the shear modulus.

Integrating

$$y = \frac{\tau_{max}}{G}x + C_7 \tag{2.37}$$

Using the condition: y = 0 at x = c, one obtains

$$C_7 = -\frac{\tau_{max}c}{G} \tag{2.38}$$

Then the deflection of point B becomes

$$y_B = \frac{\tau_{max}b}{G} + C_7 \tag{2.39}$$

(2) $a < L < x < b$

Refering to Fig. 2-9

$$r_p = x\tan\theta_p$$

$$I_2 = \frac{\pi(x^4\tan^4\theta_p - r_i^4)}{4} \tag{2.40}$$

$$b_2 = 2(r_p - r_i) \tag{2.41}$$

$$\tau_{max} = \frac{P}{I_2 b_2} \int_{r_i}^{r_p} y dA$$

$$= \frac{P(r_p^2 + r_p r_i + r_i^2)}{3I_2} \tag{2.42}$$

$$\frac{dy}{dx} = \frac{\tau_{max}}{G}$$

$$= \frac{4P(r_p^2 + r_p r_i + r_i^2)}{3\pi G(r_p^4 - r_i^4)} \tag{2.43}$$

$$dx = \frac{dr_p}{\tan\theta_p} \tag{2.44}$$

Let

$$A_8 = \frac{4P}{3\pi G \tan\theta_p} \tag{2.45}$$

$$r_{pB} = b\tan\theta_p \tag{2.46}$$

$$r_{pL} = L\tan\theta_p \tag{2.47}$$

If $r_i \neq 0$

$$y = A_8 \int \left(\frac{r_p^2 + r_i r_p + r_i^2}{r_p^4 - r_i^4}\right) dr_p + C_8$$

$$= A_8[D_5 \ln|r_p - r_i| + D_6 \ln|r_p + r_i| +$$

$$\frac{D_7}{2} \ln(r_p^2 + r_i^2)] + C_8 \qquad (2.48)$$

where

$$D_5 = \frac{3}{4r_i} \qquad (2.49)$$

$$D_6 = -\frac{1}{4r_i} \qquad (2.50)$$

$$D_7 = -\frac{1}{2r_i} \qquad (2.51)$$

Using the condition: $y = y_B$ at $x = b$, one obtains

$$C_8 = y_B - A_8[D_5 \ln|r_{pB} - r_i| + D_6 \ln|r_{pB} + r_i| +$$

$$\frac{D_7}{2} \ln(r_{pB}^2 + r_i^2)] \qquad (2.52)$$

Setting $x = L$, one obtains the deflection and slope at point L

$$y_L = A_8[D_5 \ln|r_{pL} - r_i| + D_6 \ln|r_{pL} + r_i| +$$

$$\frac{D_7}{2} \ln(r_{pL}^2 + r_i^2)] + C_8 \qquad (2.53)$$

$$y_L' = \frac{4P(r_{pL}^2 + r_{pL}r_i + r_i^2)}{3\pi G(r_{pL}^4 - r_i^4)} \qquad (2.54)$$

If $r_i = 0$

$$y = A_8 \int \frac{1}{r_p^2} dr_p + C_8'$$

$$= - \frac{A_8}{r_p} + C_8' \qquad (2.55)$$

Using the condition: $y = y_B$ at $x = b$, one obtains

$$C_8' = y_B + \frac{A_8}{r_{pB}} \qquad (2.56)$$

Setting $x = L$, the deflection and slope at point L become

$$y_L = - \frac{A_8}{r_{pL}} + C_8' \qquad (2.57)$$

$$y_L' = \frac{4P}{3\pi G r_{pL}^2} \qquad (2.58)$$

(3) $a < x < L$

Setting $x = a$ in equations (2.43) and (2.48) (or (2.55) as $r_i = 0$), one obtains y_A and y_A'

$$y_L = y_A + (L - a) y_A' \qquad (2.59)$$

$$y_L' = y_A' \qquad (2.60)$$

Using the same method disscussed in section 2.2.1, one can obtain the displacement of the grid point G by rotation about point L and translation.

2.2.3 Deflection Due to Axial Force

Referring to Fig. 2-7, one can divide the beam into 3 portions for derivation of axial deflections.

(1) $b < x < c$

The cross-sectional area of the shaft is

$$A_i = \pi(r_s^2 - r_i^2) \tag{2.61}$$

The deflection along the axis of the shaft is

$$\delta_1 = \frac{P_t(c - b)}{\pi E(r_s^2 - r_i^2)} \tag{2.62}$$

(2) $a < L < x < b$

$$r_p = x\tan\theta_p$$

The cross-sectional area of the pitch cone is

$$A_{ii} = \pi(x^2\tan^2\theta_p - r_i^2) \tag{2.63}$$

$$d\delta_2 = \frac{P_t dx}{A_{ii}E}$$

$$= \frac{P_t dx}{\pi E(x^2\tan^2\theta_p - r_i^2)}$$

$$\delta_2 = \frac{P_t}{\pi E\tan^2\theta_p} \int_L^b \frac{1}{(x^2 - \frac{r_i^2}{\tan^2\theta_p})} dx \qquad (2.64)$$

Let

$$A_9 = \frac{P_t}{\pi E\tan^2\theta_p} \qquad (2.65)$$

$$A_7 = \frac{r_i}{\tan\theta_p}$$

If $r_i \neq 0$

$$\delta_2 = A_9 \int_L^b (\frac{1}{x^2 - A_7^2})dx$$

$$= \frac{A_9}{2A_7} \ln\left|\frac{(b - A_7)(L + A_7)}{(b + A_7)(L - A_7)}\right| \qquad (2.66)$$

If $r_i = 0$

$$\delta_2 = A_9 \int_L^b \frac{dx}{x^2}$$

$$= A_9(\frac{1}{L} - \frac{1}{b}) \qquad (2.67)$$

(3) $a < x < L$

If $r_i \neq 0$

$$\delta_2 = A_9 \int_a^b \left(\frac{1}{x^2 - A_7^2}\right)dx$$

$$= \frac{A_9}{2A_7} \ln \left|\frac{(b - A_7)(a + A_7)}{(b + A_7)(a - A_7)}\right| \qquad (2.68)$$

If $r_i = 0$

$$\delta_2 = A_9 \int_a^b \frac{dx}{x^2}$$

$$= A_9\left(\frac{1}{a} - \frac{1}{b}\right) \qquad (2.69)$$

The displacement of grid point G due to the axial force is

$$\delta = \delta_1 + \delta_2 \qquad (2.70)$$

2.2.4 Deflections Due to Torsion

Again, refering to Fig. 2-7, one can divide the beam into 3 portions for derivations of torsional deflections.

(1) $b < x < e$

The polar moment of inertia of the shaft is

$$J_1 = \frac{\pi(r_s^4 - r_i^4)}{2} \tag{2.71}$$

The total angle of twist of this portion becomes

$$\phi_1 = \frac{T(e - b)}{GJ_1}$$

$$= \frac{2(e - b)T}{\pi G(r_s^4 - r_i^4)} \tag{2.72}$$

(2) $a < L < x < b$

$$r_p = x\tan\theta_p$$

$$J_2 = \frac{\pi(x^4\tan^4\theta_p - r_i^4)}{2} \tag{2.73}$$

$$d\phi_2 = \frac{Tdx}{GJ_2}$$

$$\phi_2 = \int_L^b \left[\frac{2T}{\pi G(x^4\tan^4\theta_p - r_i^4)}\right]dx$$

$$= \frac{2T}{\pi G\tan^4\theta_p}\int_L^b \left(\frac{1}{x^4 - \frac{r_i^4}{\tan^4\theta_p}}\right)dx \tag{2.74}$$

Let

$$A_{10} = \frac{2T}{\pi G \tan^4 \theta_p} \tag{2.75}$$

$$A_7 = \frac{r_i}{\tan\theta_p}$$

If $r_i \neq 0$

The total angle of twist of this portion becomes

$$\phi_2 = A_{10} \int_L^b \left(\frac{1}{x^4 - A_7^4}\right) dx$$

$$= A_{10}\left[\ln\left(\left|\frac{b - A_7}{L - A_7}\right|^{D_9} \cdot \left|\frac{b + A_7}{L + A_7}\right|^{D_{10}}\right) + \right.$$

$$\left. \frac{D_{12}}{A_7}\left(\tan^{-1}\frac{b}{A_7} - \tan^{-1}\frac{L}{A_7}\right)\right] \tag{2.76}$$

where

$$D_9 = \frac{1}{4A_7^3} \tag{2.77}$$

$$D_{10} = -\frac{1}{4A_7^3} \tag{2.78}$$

$$D_{12} = -\frac{1}{2A_7^2} \tag{2.79}$$

If $r_i = 0$

$$\Phi_2 = A_{10}\int_L^b \frac{dx}{x^4}$$

$$= \frac{A_{10}}{3}\left(\frac{1}{L^3} - \frac{1}{b^3}\right) \qquad (2.80)$$

(3) $L < x < a$

If $r_i \neq 0$

$$\Phi_2 = A_{10}\int_a^b \left(\frac{1}{x^4 - A_7^4}\right)dx$$

$$= A_{10}\left[\ln\left(\left|\frac{b - A_7}{a - A_7}\right|^{D_9} \cdot \left|\frac{b + A_7}{a + A_7}\right|^{D_{10}}\right) + \right.$$

$$\left. \frac{D_{12}}{A_7}\left(\tan^{-1}\frac{b}{A_7} - \tan^{-1}\frac{9}{A_7}\right)\right] \qquad (2.81)$$

If $r_i = 0$

$$\Phi_2 = A_{10}\int_a^b \frac{dx}{x^4}$$

$$= \frac{A_{10}}{3}\left(\frac{1}{a^3} - \frac{1}{b^3}\right) \qquad (2.82)$$

ϕ_1 and ϕ_2 can then be added to obtain the total angular displacement of the grid point G.

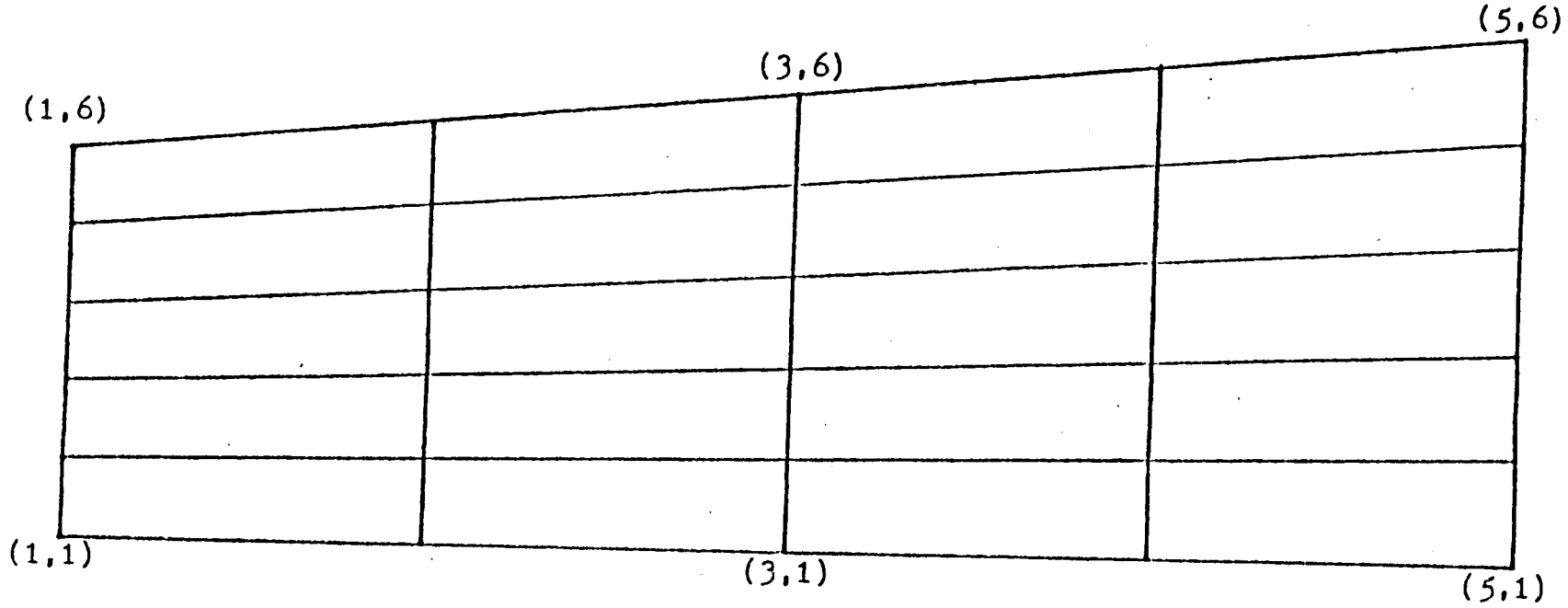

Fig. 2-1 Grid points of a tooth

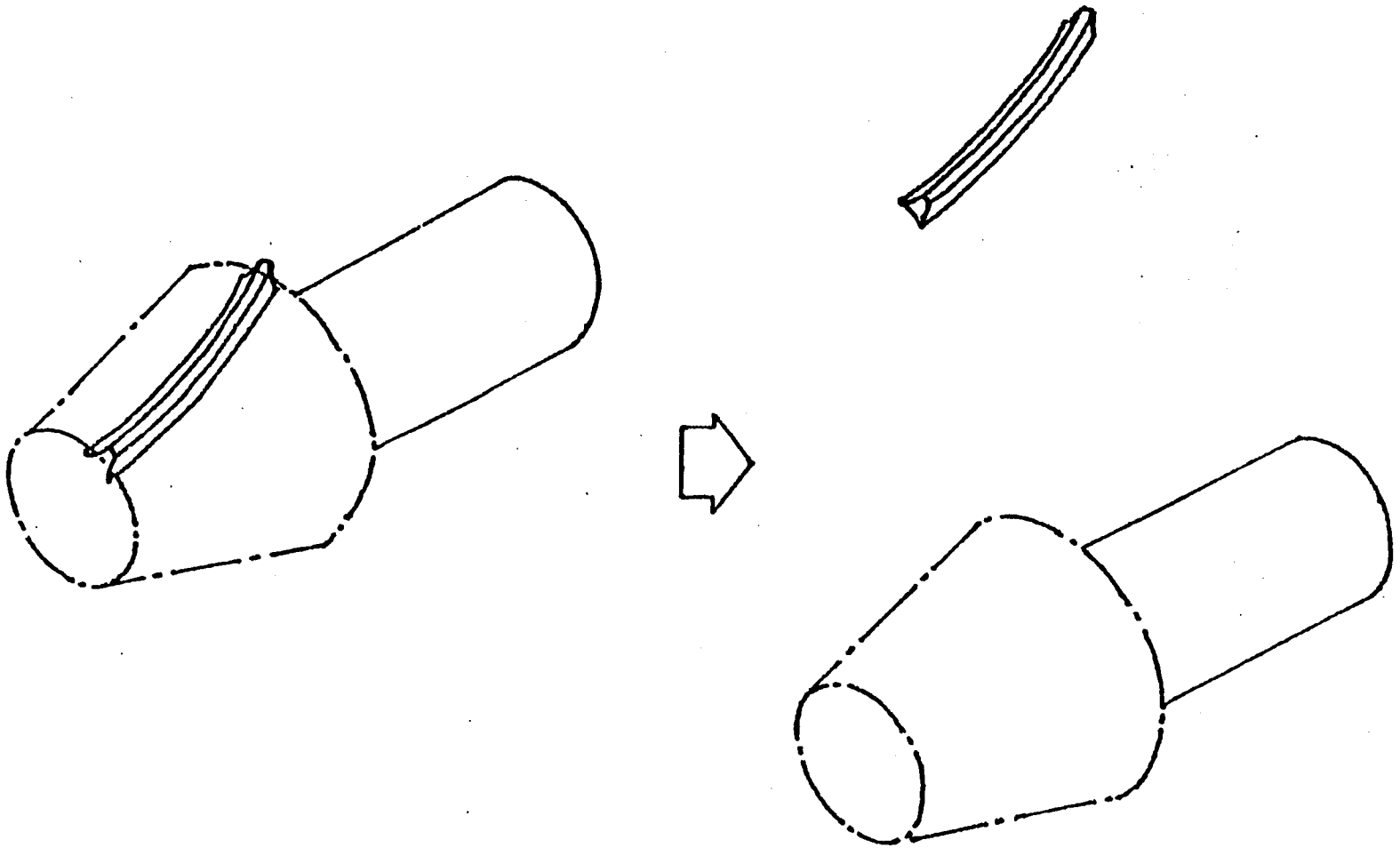

Fig. 2-2 Separation of gear

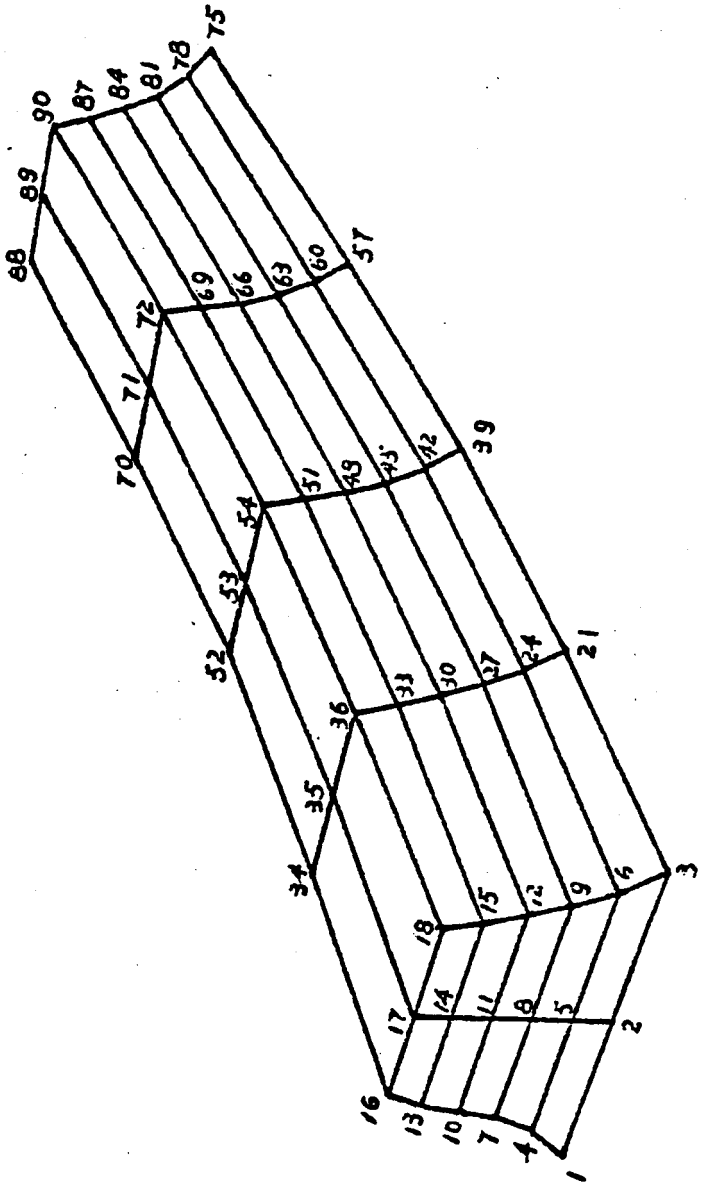

Fig. 2-3 Node number of a tooth

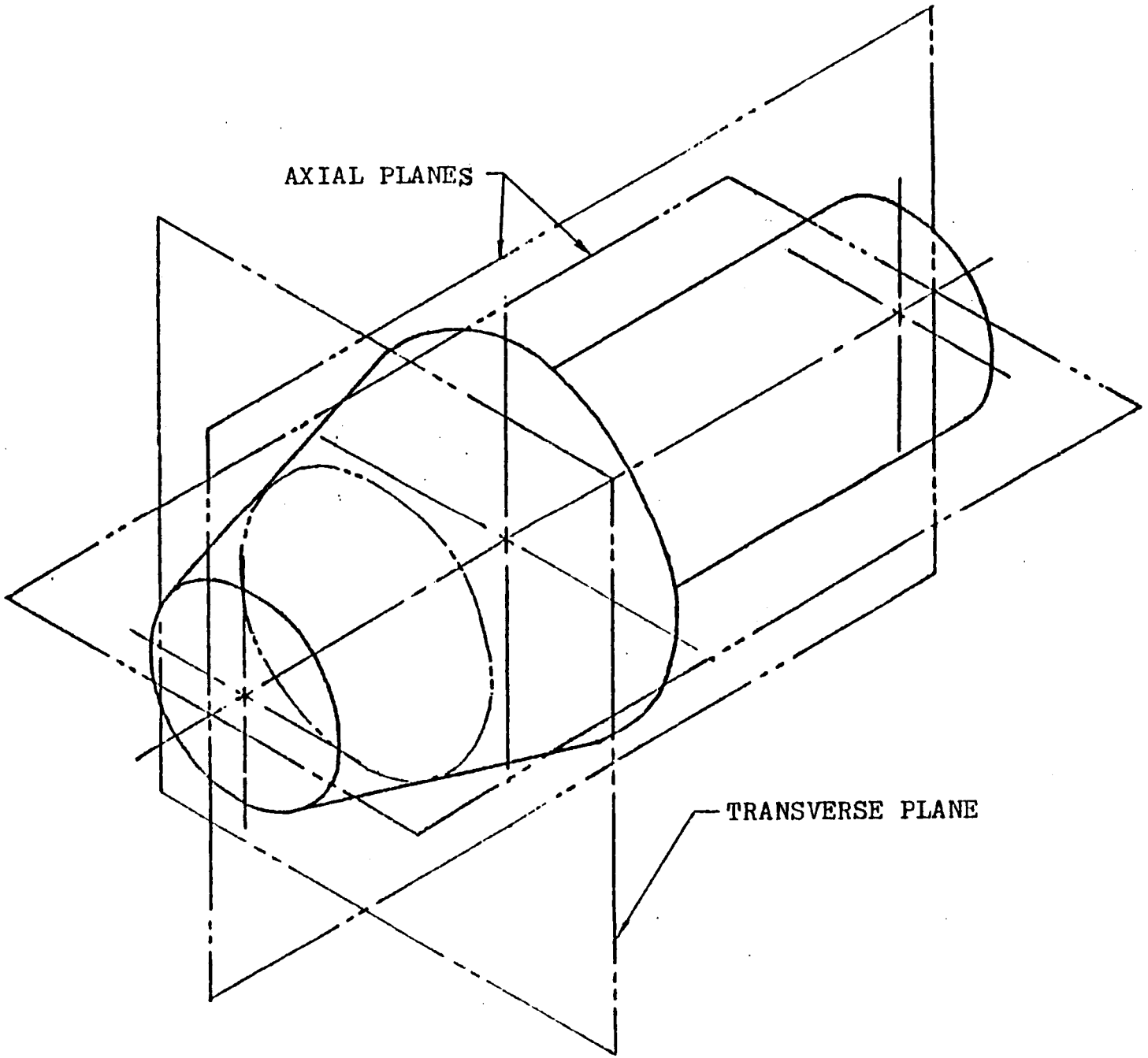

Fig. 2-4 Axial plane and transverse plane

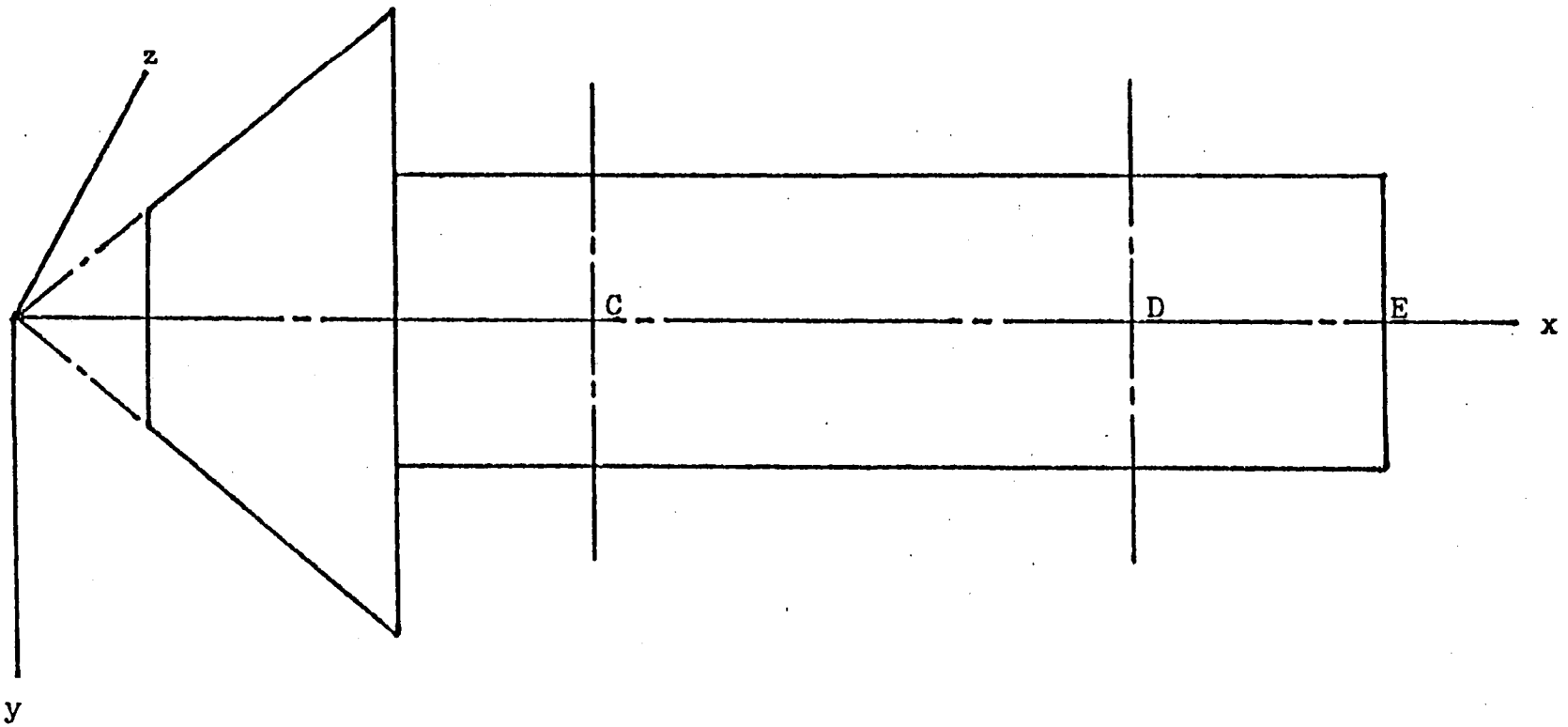

Fig. 2-5 Selection of coordinates for the calculation of deflection of wheel and shaft

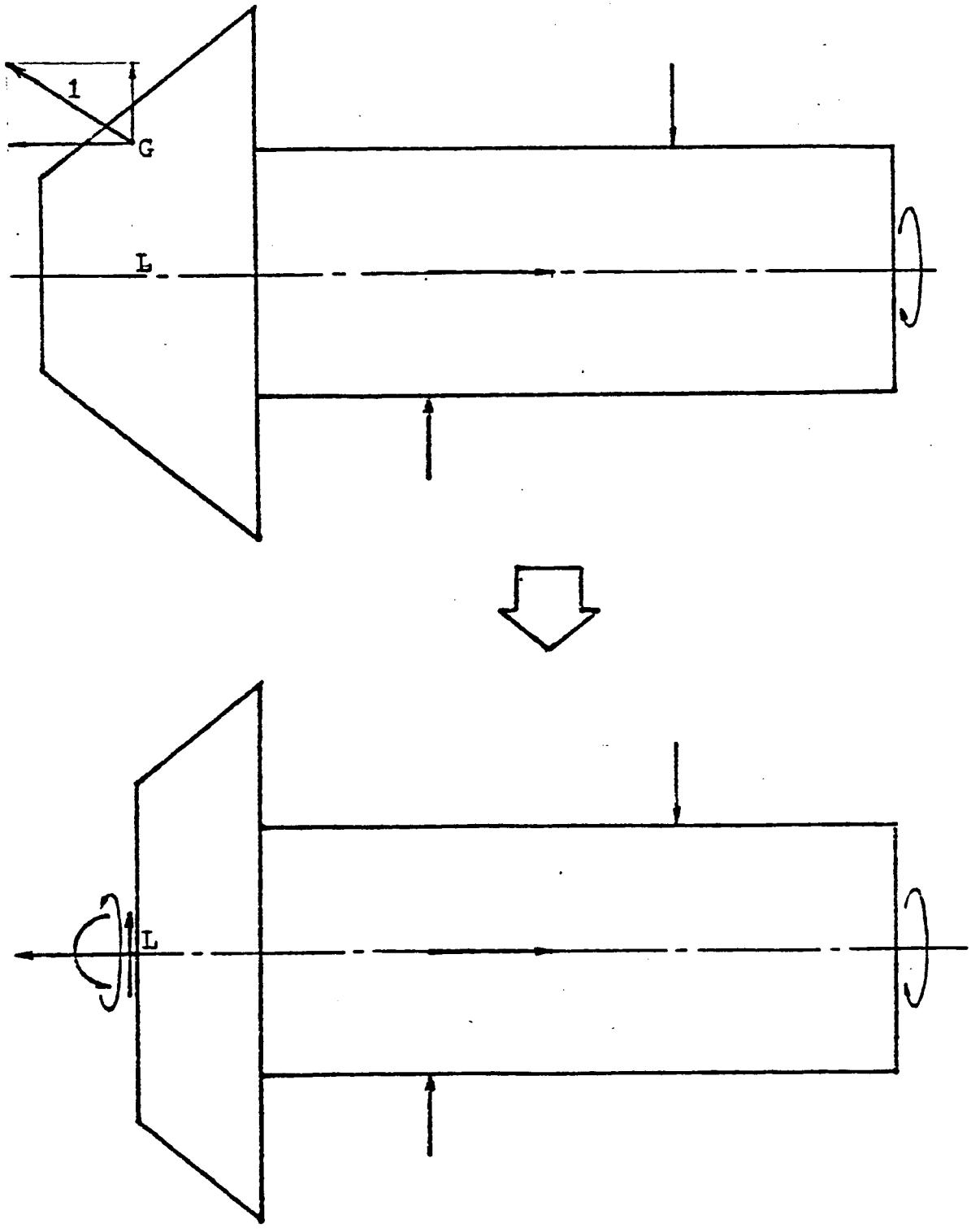

Fig. 2-6 Resolution of the unit normal force acted on a grid point

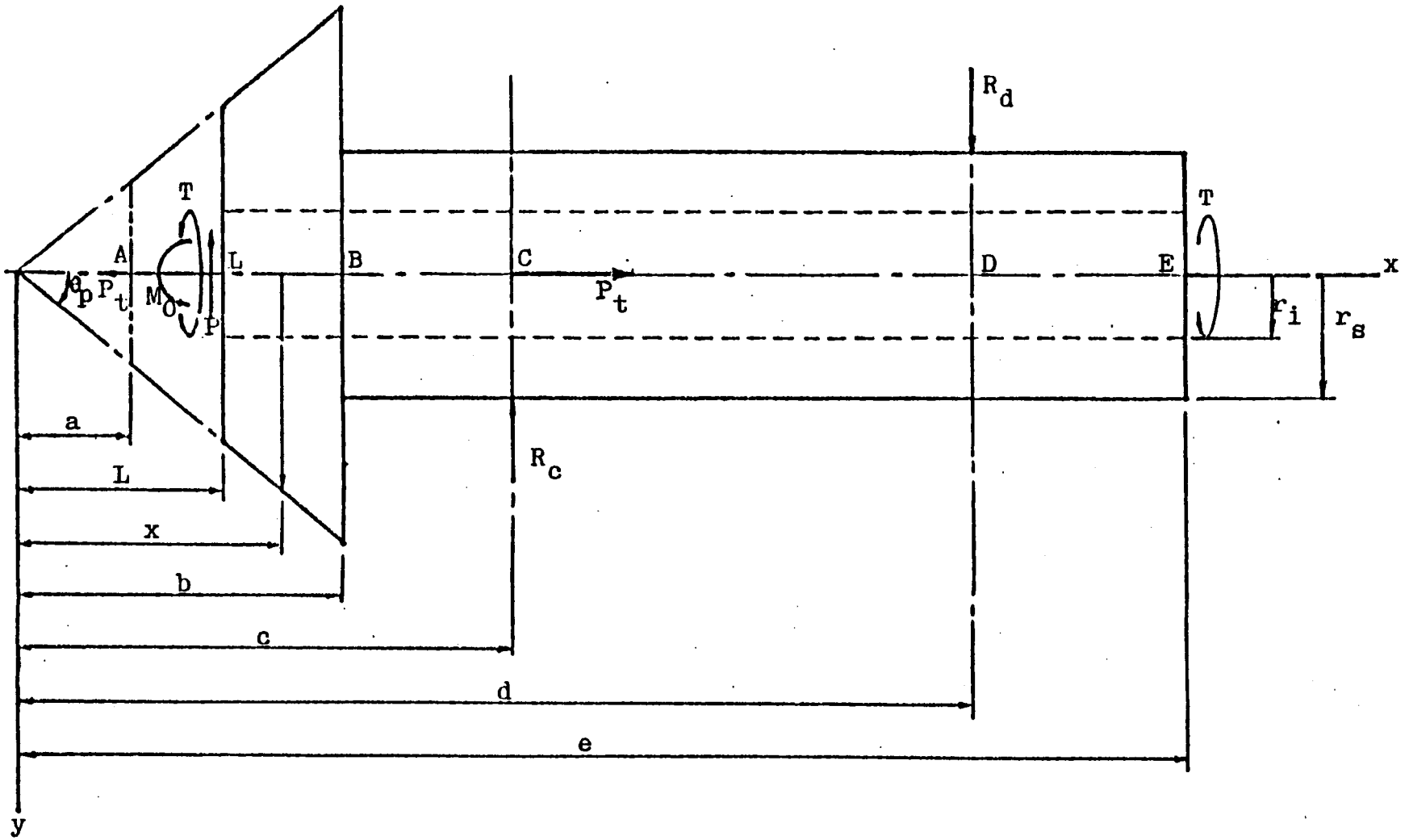

Fig. 2-7 Free body diagram after the resolution of the unit normal force

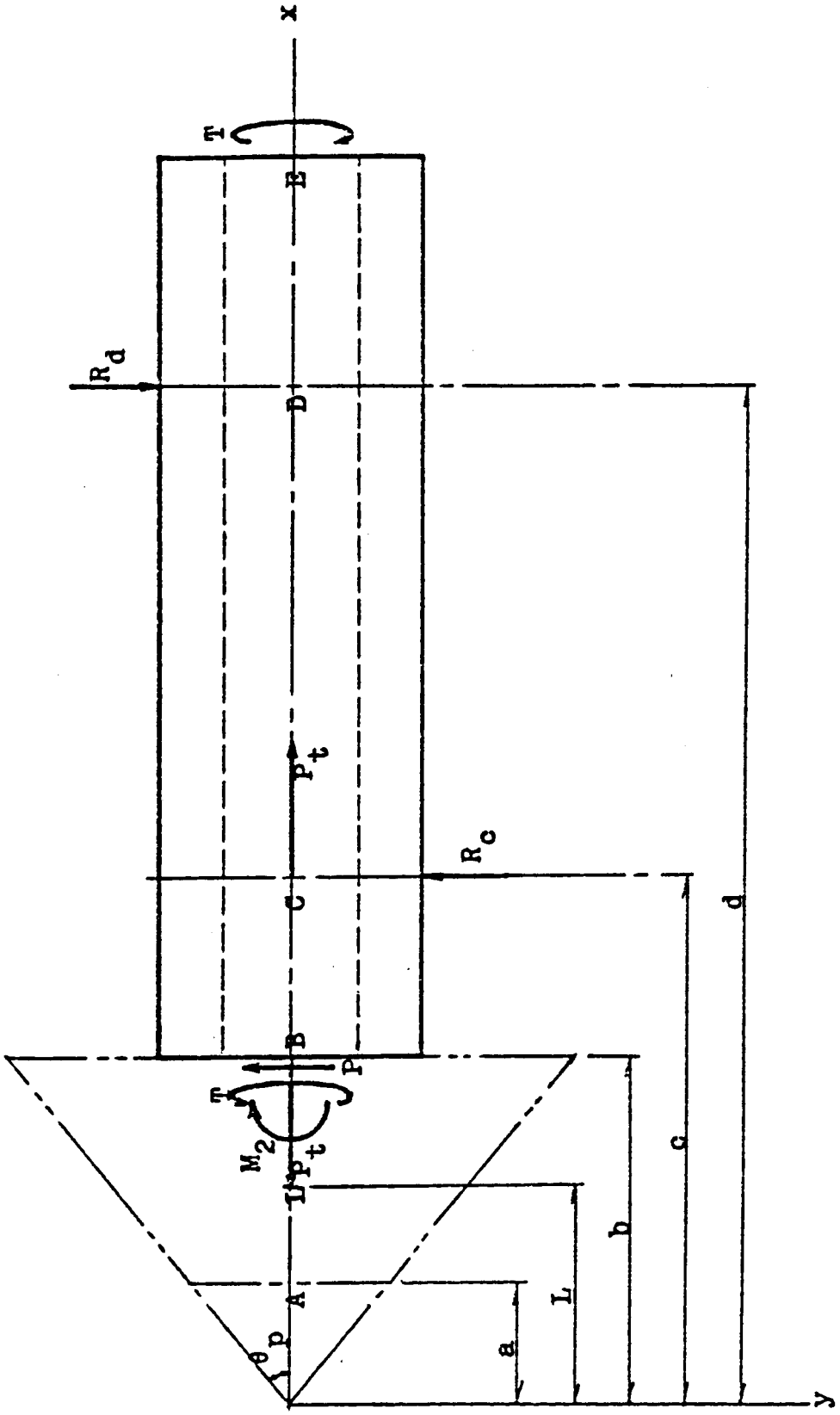

Fig. 2-8 Free body diagram of the beam between the points B and E

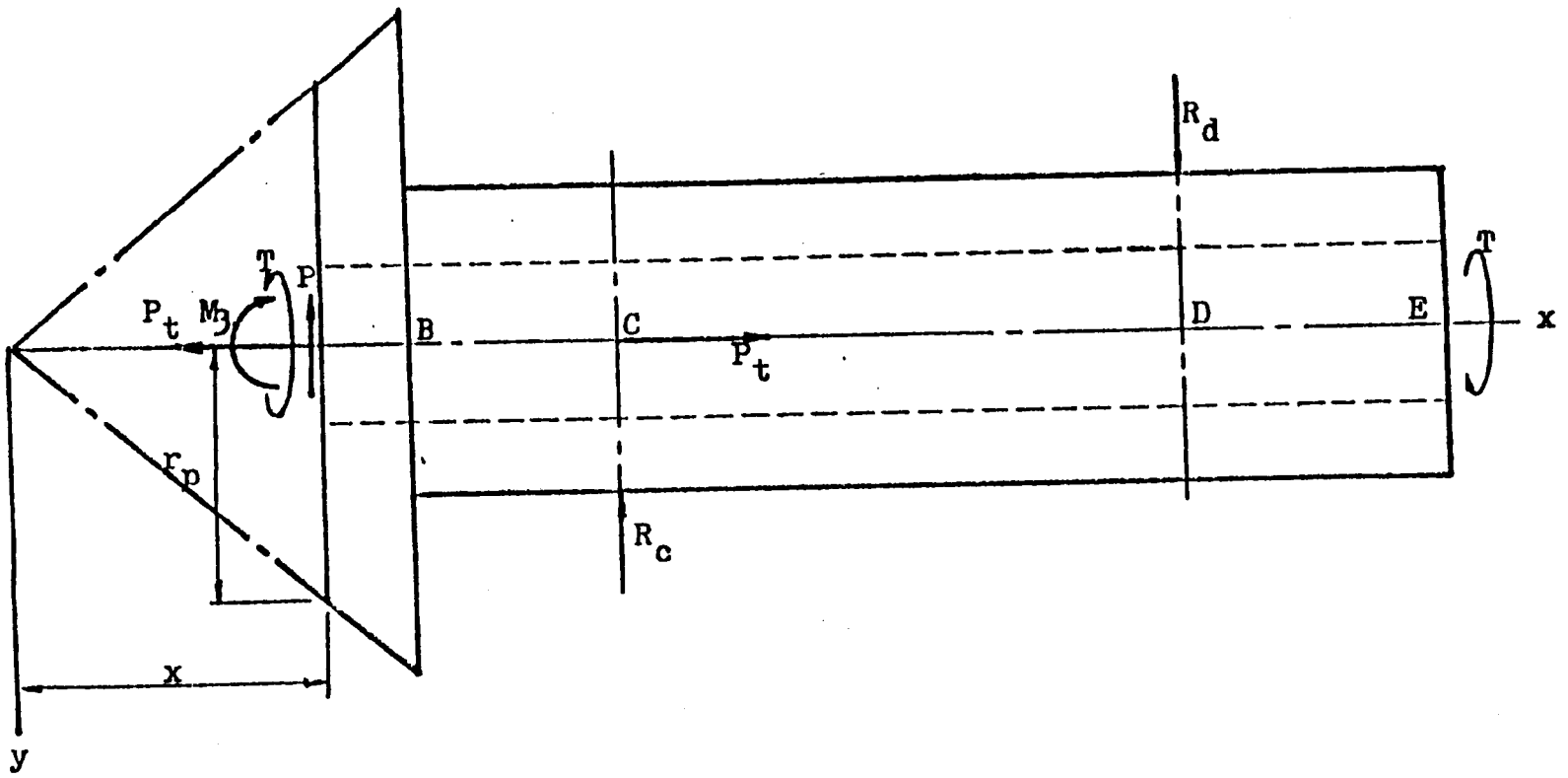

Fig. 2-9 Free body diagram of the beam to the right of point L

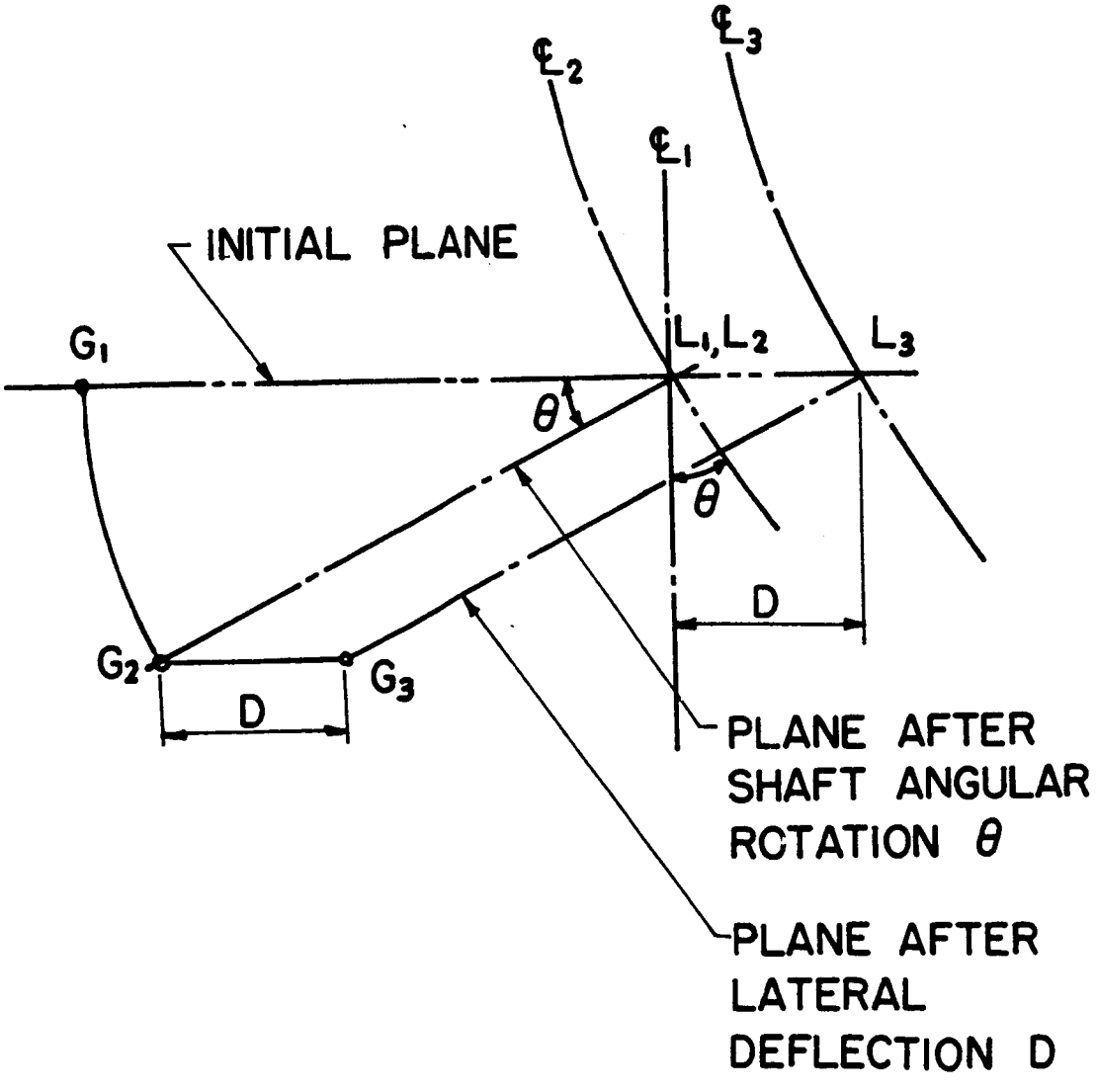

Fig. 2-10. Displacement of the grid point G

CHAPTER III

DISCUSSION OF RESULTS

3.1 Comparison with Charles Chao's Results

Fig. 3-1(p. 35) shows the variation of dynamic load factor(the ratio of the maximum dynamic load along the contact path to the static load at pitch point) as the speed of gear rotation is changed(bearing stiffness = 2,000,000 lbf/in, damping coefficient = 25 lbf.s/in). Since there are 11 degrees of freedom in the spiral bevel gear system, 11 peaks of dynamic load are expected. In the low speed region, a smaller time step in the Runge-Kutta method should be used to facilitate the convergence(Chao used H = ASTEP/WG as the time step in Runge-Kutta method, where ASTEP is the rotational angle of the gear between two adjacent contact points used in Runge-Kutta method and WG is the angular speed of the gear).

Compared with Chao's results in Fig. 3-2, one finds that the peaks in Fig. 3-1 shift toward the left side. The reason is that Chao used smaller masses(m), moments of inertia(I) and flexibility influence coefficients(a) in the gear system. From Dunkerley's Formula(11), one finds

$$\omega_n^2 < tr(\underline{M}^{-1}\underline{K}) \qquad (3.1)$$

where ω_n is the largest resonance frequency, tr means

the trace of the following matrix, $\underline{M}$ is the mass matrix and $\underline{K}$ is the stiffness matrix.

When m and I become larger $\underline{M}^{-1}$ becomes smaller. k(element in the stiffness matrix) is the reciprocal of a. So $\underline{K}$ becomes smaller as a becomes larger. In Fig. 3-1, larger m's, I's and a's are used. Then the upper limit of ω_n^2 becomes smaller. This is why the peaks in Fig. 3-1 shift toward the left side.

3.2 Effect of Mass and Moment of Inertia

The upper bond to the resonance frequencies becomes larger when the mass and moment of inertia become smaller. In Fig. 3-3, all the factors are same as those used in Fig. 3-1, except the m and I are smaller. It is seen that the peaks in Fig. 3-3 shift toward the right.

3.3 Effect of Stiffness

The upper bond to the resonance frequencies becomes larger when the stiffness becomes larger. There are two kinds of stiffness in the spiral bevel gear system - contacting stiffness and bearing stiffness. In Fig. 3-4, a larger contacting stiffness was used in comparison to that used in Fig. 3-1. In contrast, a bearing stiffness smaller than that in Fig. 3-1 was used in Fig. 3-5. Expectedly, the peaks in Fig. 3-4 moves toward right due to stiffer teeth, and the peaks in Fig. 3-5 moves toward left due to a softer bearing stiffness.

3.4 Effect of Damping Coefficient

The change of damping coefficient has little influence on the resonance frequency. But the dynamic load increases as the damping coefficient decreases. In Fig. 3-6, a damping coefficient (DC) 15 lbf.s/in was adopted instead of DC = 25 lbf.s/in in Fig. 3-1. It is clearly see that the dynamic load factor in Fig. 3-6 is considerably increased due to a smaller damping coefficient.

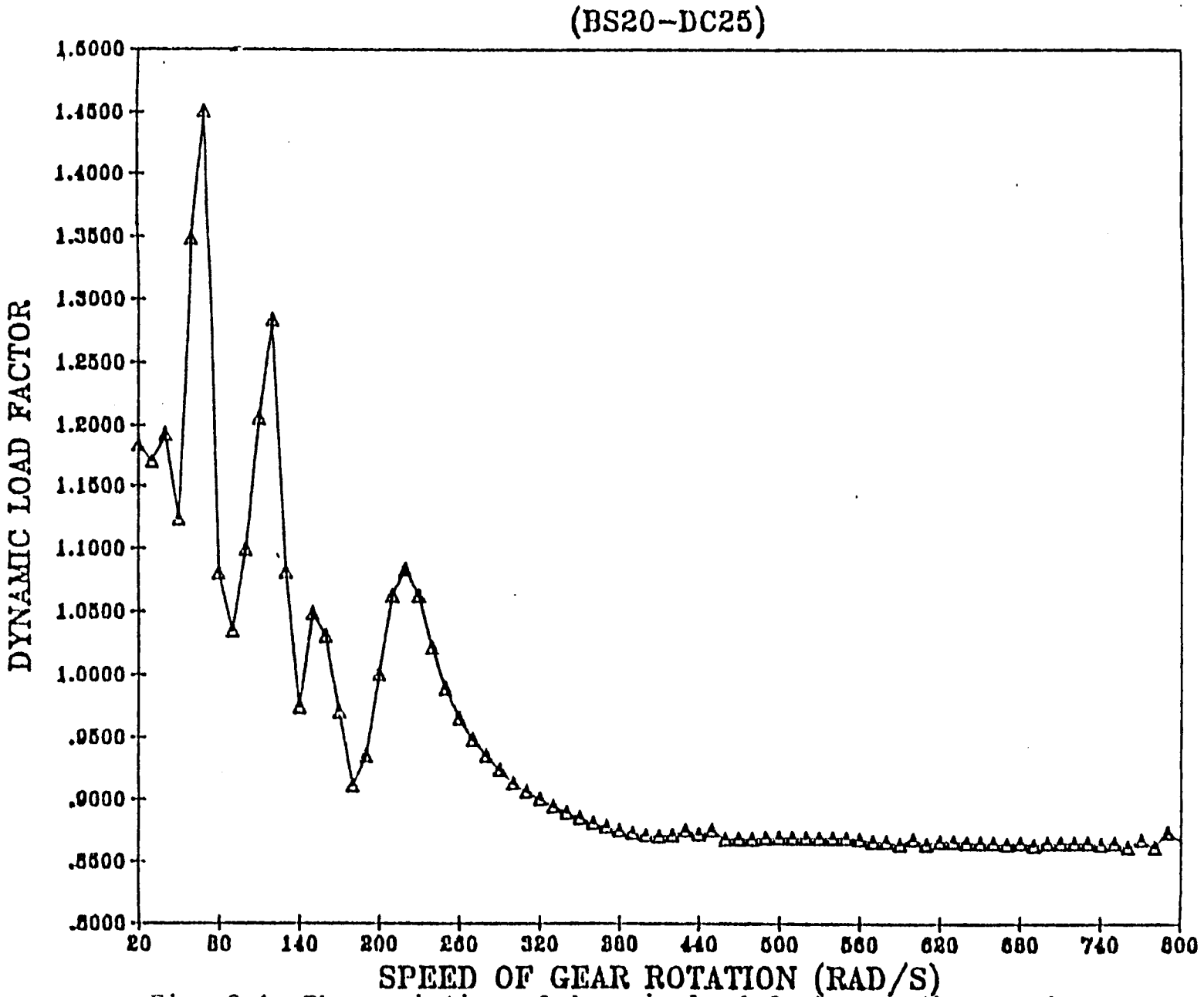

Fig. 3-1 The variation of dynamic load factor as the speed of gear rotation is changed(Hsu's m, I and a, bearing stiffness = 2,000,000 lbf/in, damping coefficient = 25 lbf.s/in)

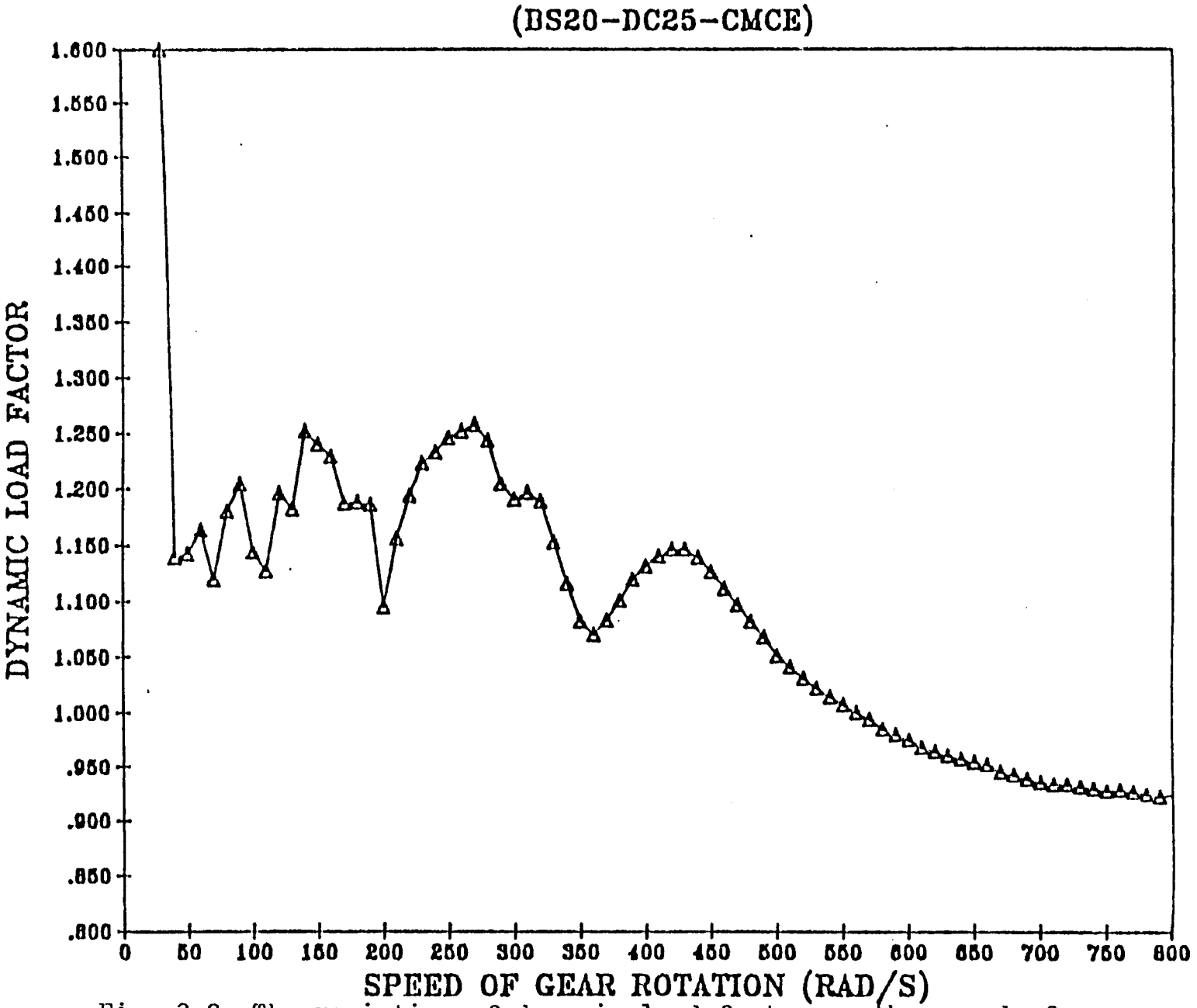

Fig. 3-2 The variation of dynamic load factor as the speed of gear rotation is changed(Chao's m, I and a, bearing stiffness = 2,000,000 lbf/in, damping coefficient = 25 lbf.s/in)

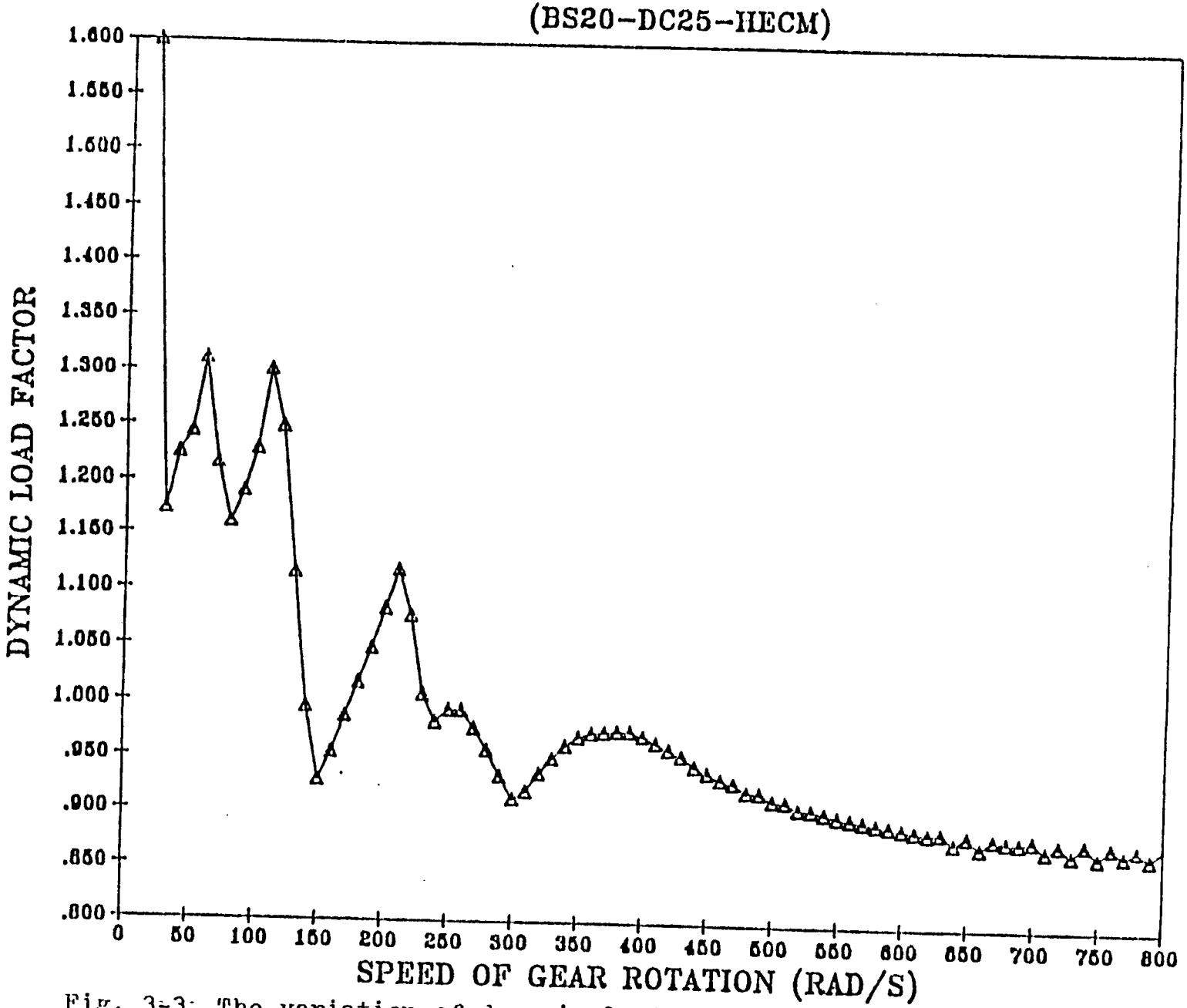

Fig. 3-3 The variation of dynamic load factor as the speed of gear rotation is changed(Chao's m and I, Hsu's a, bearing stiffness = 2,000,000 lbf/in, damping coefficient = 25 lbf.s/in)

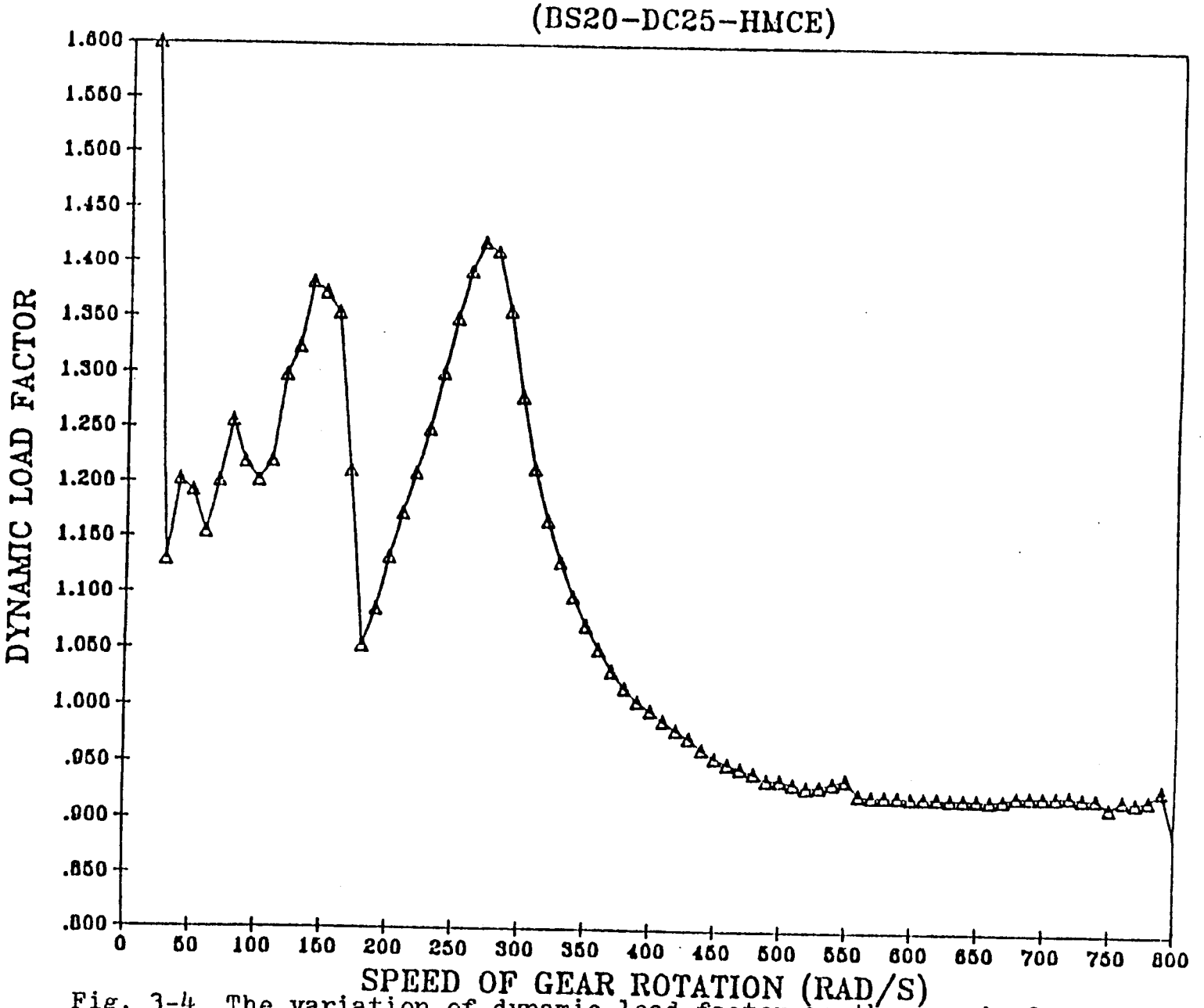

Fig. 3-4 The variation of dynamic load factor as the speed of gear rotation is changed(Hsu's m and I, Chao's a, bearing stiffness =2,000,000 lbf/in, damping coefficient = 25 lbf.s/in)

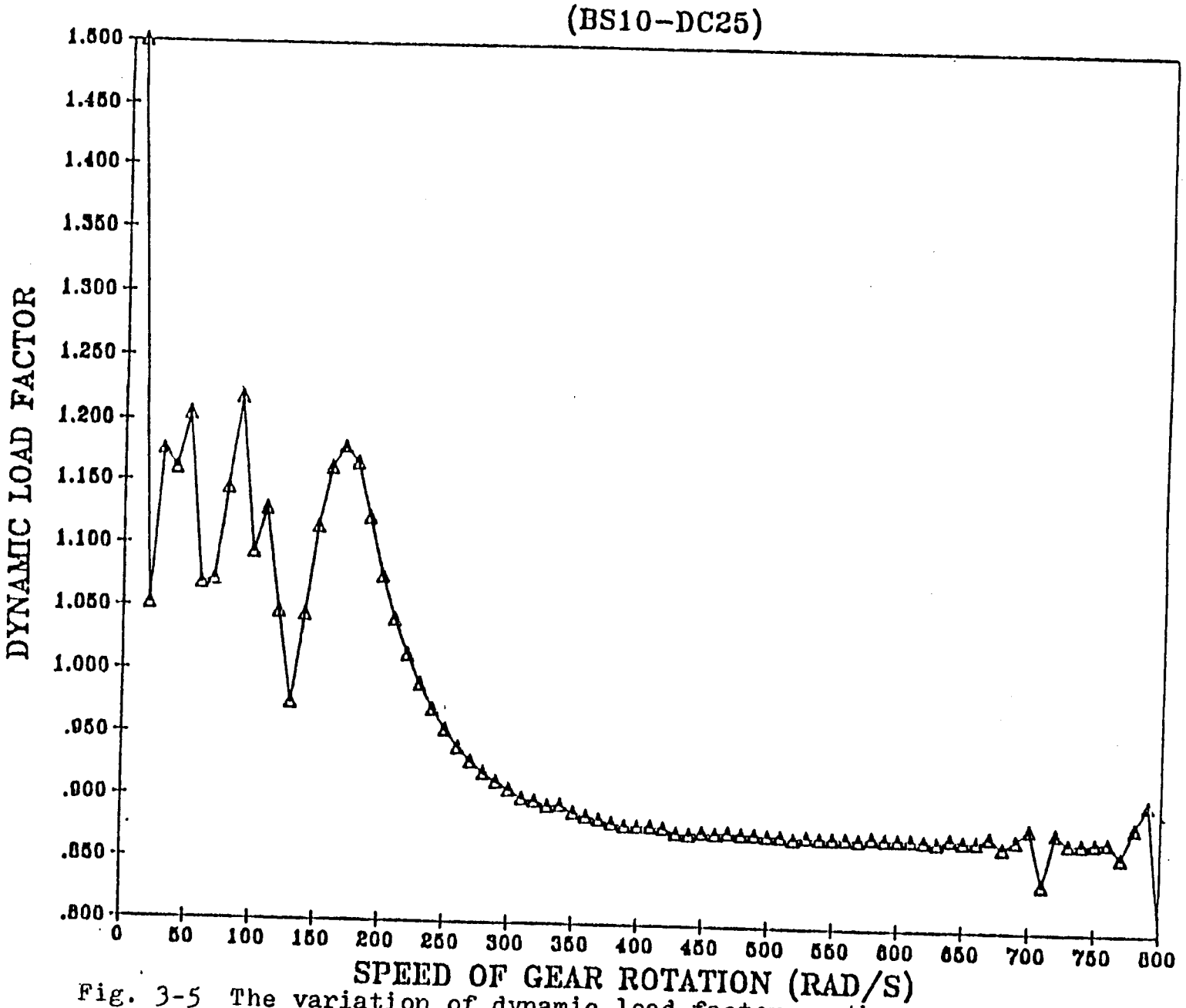

Fig. 3-5 The variation of dynamic load factor as the speed of gear rotation is changed(Hsu's m, I and a, bearing stiffness = 1,000,000 lbf/in, damping coefficient = 25 lbf.s/in)

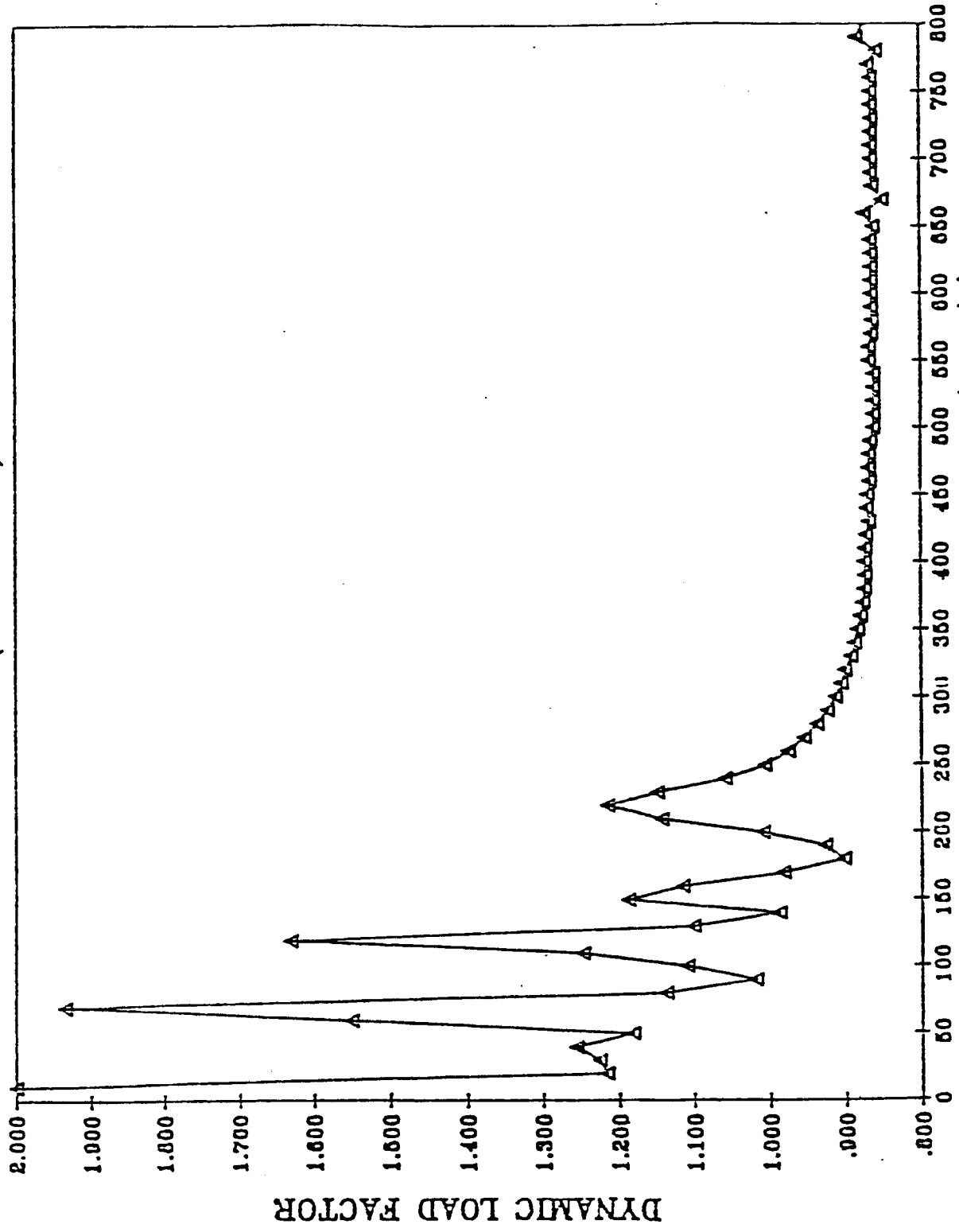

Fig. 3-6 The variation of dynamic load factor as the speed of gear rotation is changed(Ilsu's m, I and a, bearing stiffness = 2,000,000 lbf/in, damping coefficient = 15 lbf.s/in)

CHAPTER IV

CONCLUSIONS

A computer code "FLEXM" was developed to calculate the flexibility matrices of contacting teeth for spiral bevel gears using a simplified analysis based on the elementary beam theory for the deformation of gear and shaft. The simplified method requires a computer time at least one order of magnitude less than that needed for the complete FEM(Finite Element Method) Analysis reported earlier by H. C. Chao, and it is much easier to apply for different gear and shaft geometry. Results were obtained for a set of spiral bevel gears whose dimensions are shown in Fig. 4-1. In this case the teeth deflections due to torsion, bending moment, shearing strain and axial force were found to be in the order 10^{-5}, 10^{-6}, 10^{-7}, and 10^{-8} respectively. Thus, for these gears, the torsional deformation is the most predominant part.

In the analysis of dynamic load, resonance frequencies were found to be larger when the mass or moment of inertia is smaller or the stiffness is larger. The change in damping coefficient has little influence on the resonance frequency, but has a marked influence on the dynamic load at the resonant frequencies.

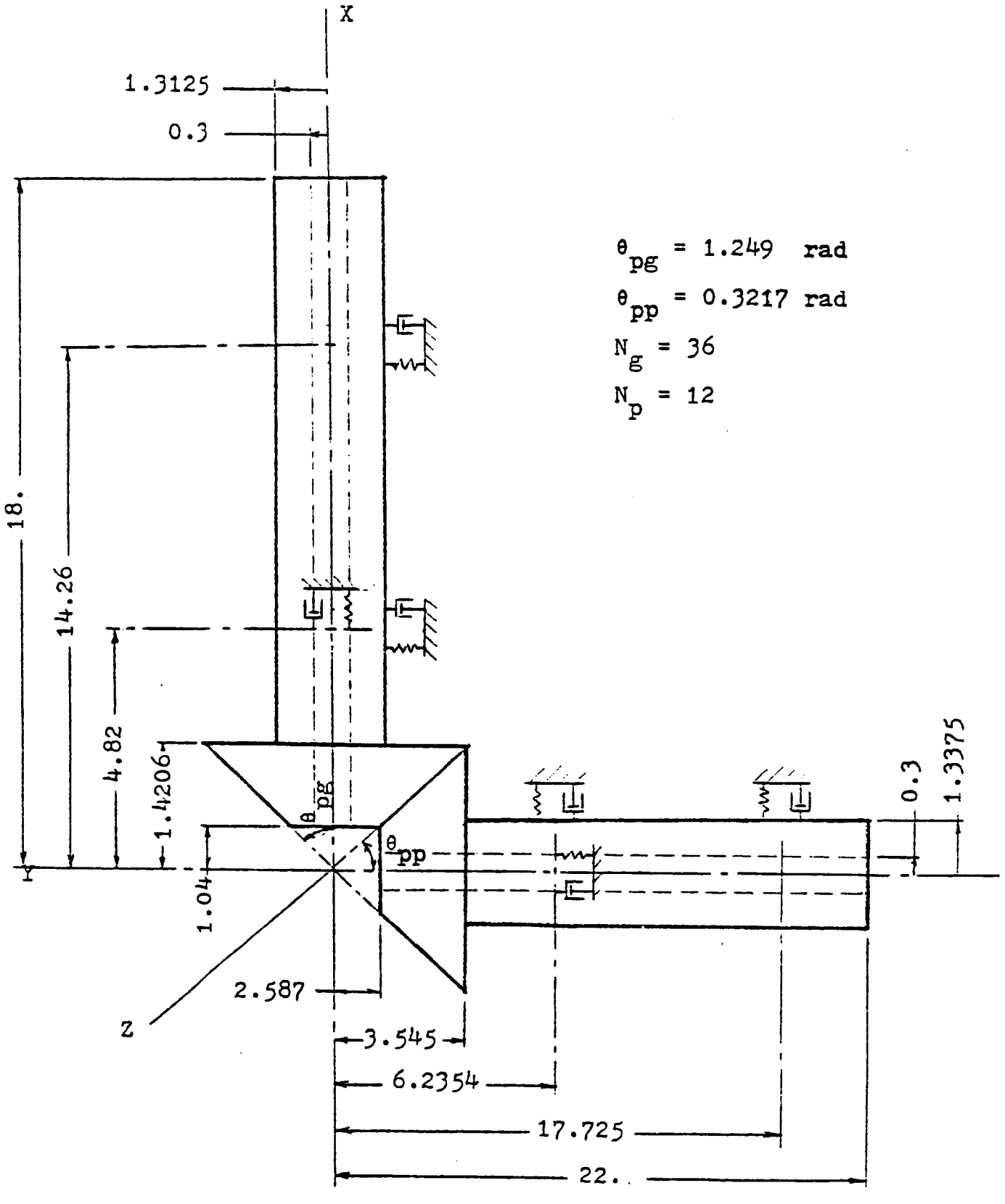

Fig. 4-1 The dimensions for a set of spiral bevel gears

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NOMENCLATURE

a(in Chapter II)	length along gear (or pinion) axis from the apex of pitch cone to the front plane (refer to Fig. 2-7)
a(in Chapter III)	flexibility influence coefficient
b	length along gear (or pinion) axis from the apex of pitch cone to the back plane (refer to Fig. 2-7)
c	length along gear (or pinion) axis from the apex of pitch cone to the center point of thrust bearing (refer to Fig. 2-7)
DC	damping coefficient
$(D_x)_C$	displacement of point C along x axis
d	length along gear (or pinion) axis from the apex of pitch cone to the center point of radial bearing (refer to Fig. 2-7)
E	modulus of elasticity
e	length along gear (or pinion) axis from the apex of pitch cone to the end plane of shaft (refer to Fig. 2-7)
G	shear modulus
I	moment of inertia
$\underline{K}$	stiffness matrix

L	length along gear (or pinion) axis from the apex of pitch cone to the transverse plane through the grid point G (refer to Fig. 2-7)
M	moment
$\underline{M}$	mass matrix
m	mass
P	shear force
P_t	force along axis of shaft
r_i	radius of hole (refer to Fig. 2-7)
r_p	radius of pitch cone on the cross section passed by transverse plane at a distance x from y axis
r_s	radius of shaft (refer to Fig. 2-7)
T	torque about x axis
y_B	deflection at B in y direction
y'_B	slope at B
δ	deflection along axis of shaft
θ_p	pitch angle
$(\theta_x)_E$	angular displacement about x axis at point E
ϕ	twist angle about x axis

τ_{max}	maximum shear stress
ω_n	the largest resonance frequency

APPENDIX A

TWO SAMPLES OF INPUT DATA FOR SAPIV

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  20 PASSWORD
  30 ATTACH,A,HSUSAP4-BI.
  40 A.
  50 CATALOG,TAPE20,HSUSAP4-GEAR-OUTFLEX,RP=999.
  60 EOR
  70 GEAR(5X6)
  80    90    1   50
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    70.000
 100     2    1    1    1    1    1    1 1.0400479-2.4734411  .2859099
    70.000
 110     3    1    1    1    1    1    1 1.0401105-2.4870416  .1187368
    70.000
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 200    12                               .8891667-2.5333526  .1864398
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 220    14                               .8387884-2.5405633  .2895019
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    70.000
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    70.000
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490    41                                1.1710277-2.9732519  .0020571
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500    42                                1.1710100-2.9679665 -.1772007
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1010       386.
1020
1030
1040
1050
1060
1070     1    1    2   20   19    4    5   23   22    2    1    3
        70.
1080     5   13   14   32   31   16   17   35   34    2    1
        70.
(S,C OR RETURN)>
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        70.
1100    10   31   32   50   49   34   35   53   52    2    1
        70.
1110    11   37   38   56   55   40   41   59   58    2    1    3
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1120    15   49   50   68   67   52   53   71   70    2    1
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1130    16   55   56   74   73   58   59   77   76    2    1    3
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        70.
1180    30   32   33   51   50   35   36   54   53    2    1
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1200    35   50   51   69   68   53   54   72   71    2    1
        70.
1210    36   56   57   75   74   59   60   78   77    2    1    3
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1220    40   68   69   87   86   71   72   90   89    2    1
        70.
1230     4    1     .1916     .4065    -.8933
1240     6    2    -.4541     .2862    -.8437
1250     7    3     .2046     .4159    -.8861
1260     9    4    -.4738     .2741    -.8369
1270    10    5     .2185     .4251    -.8784
1280    12    6    -.4903     .2639    -.8306
1290    13    7     .2324     .4339    -.8705
1300    15    8    -.5047     .2551    -.8247
1310    16    9     .2465     .4422    -.8624
1320    18   10    -.5175     .2473    -.8192
1330    22   11     .1803     .4923    -.8516
1340    24   12    -.4695     .3636    -.8046
(S,C OR RETURN)>
```

```
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
1350    25   13     .1922     .5016    -.8435
1360    27   14    -.4881     .3509    -.7992
1370    28   15     .2050     .5106    -.8350
1380    30   16    -.5035     .3402    -.7942
1390    31   17     .2178     .5191    -.8265
1400    33   18    -.5170     .3307    -.7895
1410    34   19     .2307     .5273    -.8178
1420    36   20    -.5290     .3222    -.7851
1430    40   21     .1687     .5761    -.7998
1440    42   22    -.4850     .4384    -.7567
1450    43   23     .1796     .5852    -.7908
1460    45   24    -.5024     .4252    -.7529
1470    46   25     .1911     .5940    -.7815
1480    48   26    -.5168     .4139    -.7494
1490    49   27     .2027     .6024    -.7720
1500    51   28    -.5293     .4040    -.7461
1510    52   29     .2143     .6104    -.7626
1520    54   30    -.5405     .3948    -.7429
1530    58   31     .1569     .6570    -.7374
1540    60   32    -.5008     .5096    -.6996
(S,C OR RETURN)>
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
1550    61   33     .1666     .6658    -.7273
1560    63   34    -.5169     .4961    -.6977
1570    64   35     .1769     .6744    -.7169
1580    66   36    -.5301     .4844    -.6959
1590    67   37     .1872     .6826    -.7065
1600    69   38    -.5417     .4740    -.6942
1610    70   39     .1972     .6903    -.6961
1620    72   40    -.5521     .4644    -.6925
1630    76   41     .1449     .7338    -.6637
1640    78   42    -.5168     .5763    -.6331
1650    79   43     .1535     .7424    -.6522
1660    81   44    -.5314     .5627    -.6332
1670    82   45     .1624     .7506    -.6405
1680    84   46    -.5435     .5509    -.6334
1690    85   47     .1713     .7585    -.6288
1700    87   48    -.5540     .5401    -.6335
1710    88   49     .1795     .7660    -.6173
1720    90   50    -.5637     .5300    -.6335
1730
1740
(S,C OR RETURN)>
```

```
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
1750
1760
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1800
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1820
1830
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1850
1860
1870
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1890
1900
1910
1920
1930
1940
(S,C OR RETURN)>
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
1950
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
2110
2120
2130
2140
(S,C OR RETURN)>
```

```
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
2150
2160
2170
2180
2190
2200
2210
2220
2230
2240
2250
2260 EOI
>
```

```
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
 10 VCHANG,CHUF5341-9079,CM300000,T500,P0.
 20 PASSWORD
 30 ATTACH,A,HSUSAP4-BI.
 40 A.
 50 CATALOG,TAPE20,HSUSAP4-PINION-OUTFLEX,RP=999.
 60 EOR
 70 PINION(5X6)
 80    90    1   50
 90     1    1    1    1    1    1    1  .7493591-2.5867077  .0961353
    70.000
100     2    1    1    1    1    1    1  .7105503-2.5867380  .2547510
    70.000
110     3    1    1    1    1    1    1  .6396983-2.5867683  :4025284
    70.000
120     4                                 .7950604-2.5697191  .1267436
    70.000
130     5                                 .7586351-2.5697031  .2695487
    70.000
140     6                                 .6967888-2.5696870  .4033212
    70.000
(S,C OR RETURN)>
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
150     7                                 .8406950-2.5527553  .1573071
    70.000
160     8                                 .8067394-2.5526930  .2840528
    70.000
170     9                                 .7537959-2.5526307  .4041128
    70.000
180    10                                 .8836706-2.5356485  .1971727
    70.000
190    11                                 .8546212-2.5356184  .2988727
    70.000
200    12                                 .8139109-2.5355884  .3964881
    70.000
210    13                                 .9236519-2.5185588  .2447413
    70.000
220    14                                 .9026101-2.5185250  .3135165
    70.000
230    15                                 .8764554-2.5184911  .3805130
    70.000
240    16                                 .9601221-2.5014674  .2991586
    70.000
(S,C OR RETURN)>
```

```
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
250     17                                  .9507147-2.5014348  .3277974
    70.000
260     18                                  .9404533-2.5014022  .3561410
    70.000
270     19    1     1     1     1     1     1  .8276093-2.8262767 -.0357977
    70.000
280     20    1     1     1     1     1     1  .8148691-2.8262729  .1444194
    70.000
290     21    1     1     1     1     1     1  .7648936-2.8262690  .3186656
    70.000
300     22                                  .3821774-2.8076628 -.0104465
    70.000
310     23                                  .8690964-2.8076599  .1515968
    70.000
320     24                                  .8265055-2.8076571  .3084850
    70.000
330     25                                  .9366826-2.7890703  .0148756
    70.000
340     26                                  .9233059-2.7890684  .1584728
    70.000
(S,C OR RETURN)>
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
350     27                                  .8880466-2.7890665  .2983160
    70.000
360     28                                  .9903133-2.7705973  .0504391
    70.000
370     29                                  .9775167-2.7705719  .1656822
    70.000
380     30                                  .9513575-2.7705465  .2786274
    70.000
390     31                                 1.0418918-2.7519397  .0948586
    70.000
400     32                                 1.0317557-2.7519091  .1726900
    70.000
410     33                                 1.0158144-2.7518785  .2495356
    70.000
420     34                                 1.0907050-2.7333002  .1470639
    70.000
430     35                                 1.0859416-2.7332657  .1791881
    70.000
440     36                                 1.0802322-2.7332312  .2111591
    70.000
(S,C OR RETURN)>
```

```
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
450    37    1    1    1    1    1    1  .8800131-3.0656536 -.1932156
    70.000
460    38    1    1    1    1    1    1  .9001061-3.0657021  .0036763
    70.000
470    39    1    1    1    1    1    1  .8786765-3.0657506  .2010801
    70.000
480    40                                 .9431940-3.0456198 -.1753063
    70.000
490    41                                 .9593132-3.0456258  .0013620
    70.000
500    42                                 .9426273-3.0456318  .1779714
    70.000
510    43                                1.0063188-3.0256037 -.1574129
    70.000
520    44                                1.0184613-3.0255673 -.0012654
    70.000
530    45                                1.0065213-3.0255309  .1548833
    70.000
540    46                                1.0700859-3.0052346 -.1281752
    70.000
(S,C OR RETURN)>
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
550    47                                1.0776598-3.0053157 -.0034644
    70.000
560    48                                1.0707496-3.0053967  .1212768
    70.000
570    49                                1.1333126-2.9852888 -.0896833
    70.000
580    50                                1.1367420-2.9852550 -.0060019
    70.000
590    51                                1.1340015-2.9852211  .0776976
    70.000
600    52                                1.1952105-2.9650718 -.0420047
    70.000
610    53                                1.1958550-2.9650548 -.0089431
    70.000
620    54                                1.1955852-2.9650379  .0241219
    70.000
630    55    1    1    1    1    1    1  .8995507-3.3051870 -.3725657
    70.000
640    56    1    1    1    1    1    1  .9584656-3.3051394 -.1655958
    70.000
(S,C OR RETURN)>
```

```
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
650    57    1    1    1    1    1    1  .9728761-3.3050918  .0498089
    70.000
660    58                                  .9703256-3.2835967 -.3641763
    70.000
670    59                                 1.0208089-3.2835200 -.1791273
    70.000
680    60                                 1.0363269-3.2834433  .0120539
    70.000
690    61                                 1.0410536-3.2620206 -.3557925
    70.000
700    62                                 1.0830462-3.2619149 -.1929779
    70.000
710    63                                 1.0997358-3.2618091 -.0256761
    70.000
720    64                                 1.1142966-3.2402622 -.3357601
    70.000
730    65                                 1.1452837-3.2401834 -.2063931
    70.000
740    66                                 1.1613041-3.2401047 -.0743408
    70.000
(S,C OR RETURN)>
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
750    67                                 1.1888861-3.2186494 -.3055537
    70.000
760    68                                 1.2075308-3.2185239 -.2200953
    70.000
770    69                                 1.2200413-3.2183983 -.1335330
    70.000
780    70                                 1.2634673-3.1968262 -.2654222
    70.000
790    71                                 1.2695847-3.1967588 -.2342216
    70.000
800    72                                 1.2749318-3.1966915 -.2028807
    70.000
810    73    1    1    1    1    1    1  .8787037-3.5446358 -.5682845
    70.000
820    74    1    1    1    1    1    1  .9813550-3.5448013 -.3600641
    70.000
830    75    1    1    1    1    1    1 1.0383987-3.5449667 -.1342854
    70.000
840    76                                  .9556396-3.5215521 -.5715815
    70.000
(S,C OR RETURN)>
```

```
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
850    77                                1.0444118-3.5215666 -.3862172
    70.000
860    78                                1.0976054-3.5215812 -.1876949
    70.000
870    79                                1.0325396-3.4984790 -.5748770
    70.000
880    80                                1.1073122-3.4983428 -:4126808
    70.000
890    81                                1.1567846-3.4982065 -.2410796
    70.000
900    82                                1.1142402-3.4751546 -.5663933
    70.000
910    83                                1.1703313-3.4750532 -.4387410
    70.000
920    84                                1.2118531-3.4749518 -.3056425
    70.000
930    85                                1.1990483-3.4518771 -.5473019
    70.000
940    86                                1.2332332-3.4517915 -.4650465
    70.000
(S,C OR RETURN)>
    123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
    56789
950    87                                1.2617830-3.4517060 -.3806732
    70.000
960    88                                1.2857970-3.4286330 -.5179480
    70.000
970    89                                1.2959437-3.4285464 -.4918856
    70.000
980    90                                1.3055622-3.4284599 -.4656248
    70.000
990     5   40    1
1000    1 30000000.         .3      .283  .0000073
1010      386.
1020
1030
1040
1050
1060
1070     1    1    2   20   19    4    5   23   22    2    1    3
        70.
1080     5   13   14   32   31   16   17   35   34    2    1
        70.
(S,C OR RETURN)>
```

```
      123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
      56789
1090     6   19   20   38   37   22   23   41   40    2    1    3
        70.
1100    10   31   32   50   49   34   35   53   52    2    1
        70.
1110    11   37   38   56   55   40   41   59   58    2    1    3
        70.
1120    15   49   50   68   67   52   53   71   70    2    1
        70.
1130    16   55   56   74   73   58   59   77   76    2    1    3
        70.
1140    20   67   68   86   85   70   71   89   88    2    1
        70.
1150    21    2    3   21   20    5    6   24   23    2    1    3
        70.
1160    25   14   15   33   32   17   18   36   35    2    1
        70.
1170    26   20   21   39   38   23   24   42   41    2    1    3
        70.
1180    30   32   33   51   50   35   36   54   53    2    1
        70.
(S,C OR RETURN)>
      123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
      56789
1190    31   38   39   57   56   41   42   60   59    2    1    3
        70.
1200    35   50   51   69   68   53   54   72   71    2    1
        70.
1210    36   56   57   75   74   59   60   78   77    2    1    3
        70.
1220    40   68   69   87   86   71   72   90   89    2    1
        70.
1230     4    1     .2459     .5216    -.8170
1240     6    2    -.6033    -.7862     .1338
1250     7    3     .3740     .5638    -.7364
1260     9    4    -.5703    -.8077     .1497
1270    10    5     .4556     .5940    -.6631
1280    12    6    -.2475     .2416    -.9383
1290    13    7     .5149     .6190    -.5930
1300    15    8    -.3479     .2139    -.9128
1310    16    9     .5594     .6410    -.5256
1320    18   10    -.4396     .1876    -.8784
1330    22   11     .1002     .5567    -.8246
1340    24   12    -.6334    -.7652     .1154
(S,C OR RETURN)>
```

```
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
1350    25   13     .2328     .5959    -.7686
1360    27   14    -.6025    -.7875     .1297
1370    28   15     .3222     .6243    -.7117
1380    30   16    -.4116     .2801    -.8672
1390    31   17     .3899     .6477    -.6546
1400    33   18    -.5045     .2536    -.8253
1410    34   19     .4433     .6682    -.5975
1420    36   20    -.5874     .2286    -.7763
1430    40   21    -.0425     .5924    -.8045
1440    42   22    -.3572     .3740    -.8559
1450    43   23     .0905     .6285    -.7725
1460    45   24    -.4680     .3453    -.8134
1470    46   25     .1845     .6548    -.7329
1480    48   26    -.5621     .3190    -.7630
1490    49   27     .2580     .6767    -.6896
1500    51   28    -.6436     .2943    -.7065
1510    52   29     .3183     .6958    -.6439
1520    54   30    -.7150     .2708    -.6446
1530    58   31    -.1771     .6284    -.7574
1540    60   32    -.5153     .4086    -.7533
(S,C OR RETURN)>
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
1550    61   33    -.0481     .6613    -.7486
1560    63   34    -.6112     .3828    -.6927
1570    64   35     .0468     .6855    -.7265
1580    66   36    -.6909     .3588    -.6276
1590    67   37     .1239     .7057    -.6976
1600    69   38    -.7585     .3360    -.5584
1610    70   39     .1887     .7233    -.6643
1620    72   40    -.8157     .3141    -.4859
1630    76   41    -.2984     .6649    -.6848
1640    78   42    -.6517     .4435    -.6153
1650    79   43    -.1779     .6943    -.6974
1660    81   44    -.7286     .4209    -.5404
1670    82   45    -.0850     .7163    -.6926
1680    84   46    -.7911     .3994    -.4633
1690    85   47    -.0073     .7347    -.6784
1700    87   48    -.8419     .3787    -.3844
1710    88   49     .0598     .7506    -.6580
1720    90   50    -.8826     .3587    -.3040
1730
1740
(S,C OR RETURN)>
```

```
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
1750
1760
1770
1780
1790
1800
1810
1820
1830
1840
1850
1860
1870
1880
1890
1900
1910
1920
1930
1940
(S,C OR RETURN)>
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
1950
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
2110
2120
2130
2140
(S,C OR RETURN)>
```

```
     123456789 123456789 123456789 123456789 123456789 123456789 123456789 1234
     56789
2150
2160
2170
2180
2190
2200
2210
2220
2230
2240
2250     0
2260 EOI
>
```

APPENDIX B

DEFLECTION AND SLOPE AT POINT B

Referring to Fig. 2-7, one obtains the reaction force at c from $\Sigma (M_z)_C = 0$:

$$R_C = \frac{M_0 - P(d - L)}{d - c} \tag{B.1}$$

where $(M_z)_C$ is the moment about z axis through point C.

To obtain the deflection and slope at B, the beam is divide into 2 portions.

(1) $c < x < d$

Considering the equilibrium of the portion of the beam to the right of point C (Fig. B-1), one can obtain

$$M_1 = (P + R_C)x - PL - R_C c - M_0 \tag{B.2}$$

The moment of inertia of the cross sectional area with respect to the neutral axis is

$$I_1 = \frac{\pi(r_s^4 - r_i^4)}{4} \tag{B.3}$$

Let E be the modulus of elasticity, the differential equation of the elastic line for the beam is

$$y'' = -\frac{M_1}{EI_1}$$

$$= -\frac{[(P + R_C)x - (PL + R_C c + M_0)]}{EI_1} \qquad (B.4)$$

Let

$$A_1 = -\frac{(P + R_C)}{EI_1} \qquad (B.5)$$

$$A_2 = \frac{(PL + R_C c + M_0)}{EI_1} \qquad (B.6)$$

Then

$$y'' = A_1 x + A_2 \qquad (B.7)$$

Separating variables and integrating twice one obtains

$$y = \frac{A_1}{6}x^3 + \frac{A_2}{2}x^2 + C_1 x + C_2 \qquad (B.8)$$

where C_1 and C_2 are constants of integration.

The integration constants are determined from the conditions:

$y = 0$ at $x = c$ and $x = d$

Substituting these conditions into the equation (B.8), one obtains

$$C_1 = -\frac{1}{6}[A_1(c^2 + cd + d^2) + 3A_2(c + d)] \qquad (B.9)$$

$$C_2 = \frac{cd}{6}[A_1(c + d) + 3A_2] \qquad (B.10)$$

With these values, the slope at $x = c$ can be obtained

$$y'_C = \frac{c}{2}(A_1 c + 2A_2) + C_1 \qquad (B.11)$$

(2) $b < x < c$

Fig. B-2 is a free body diagram of the portion of the beam to the right of point B. One obtains

$$M_2 = P(x - L) - M_0 \qquad (B.12)$$

$$I_2 = I_1 = \frac{\pi(r_s^4 - r_i^4)}{4}$$

The differential equation is

$$y'' = -\frac{M_2}{EI_1}$$

$$= \frac{-Px + (PL + M_0)}{EI_1} \qquad (B.13)$$

Let

$$A_3 = -\frac{P}{EI_1} \qquad (B.14)$$

$$A_4 = \frac{PL + M_0}{EI_1} \tag{B.15}$$

Then

$$y'' = A_3x + A_4 \tag{B.16}$$

$$y' = \frac{A_3}{2}x^2 + A_4x + C_3 \tag{B.17}$$

$$y = \frac{A_3}{6}x^3 + \frac{A_4}{2}x^2 + C_3x + C_4 \tag{B.18}$$

At $x = c$, $y = 0$ and $y' = y'_C = \frac{c}{2}(A_1c + 2A_2) + C_1$, one obtains

$$C_3 = y'_C - \frac{c}{2}(A_3c + 2A_4) \tag{B.19}$$

$$C_4 = -\frac{c}{6}(A_3c^2 + 3A_4c + 6C_3) \tag{B.20}$$

Then the deflection and slope of point B can be obtained

$$y_B = \frac{b}{6}(A_3b^2 + 3A_4b + 6C_3) + C_4 \tag{B.21}$$

$$y'_B = \frac{b}{2}(A_3b + 2A_4) + C_3 \tag{B.22}$$

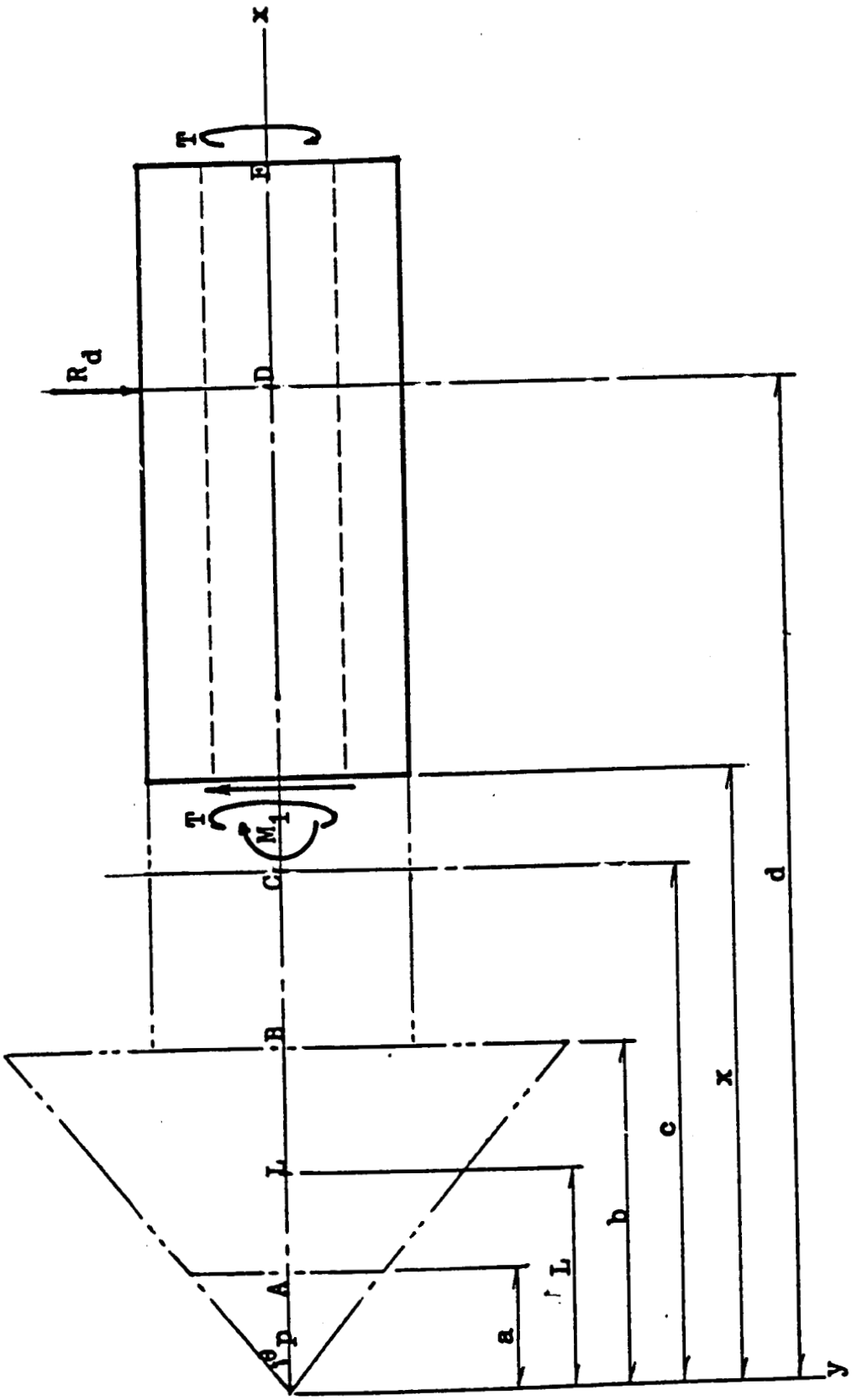

Fig. B-1 Free body diagram of the beam to the right of point C

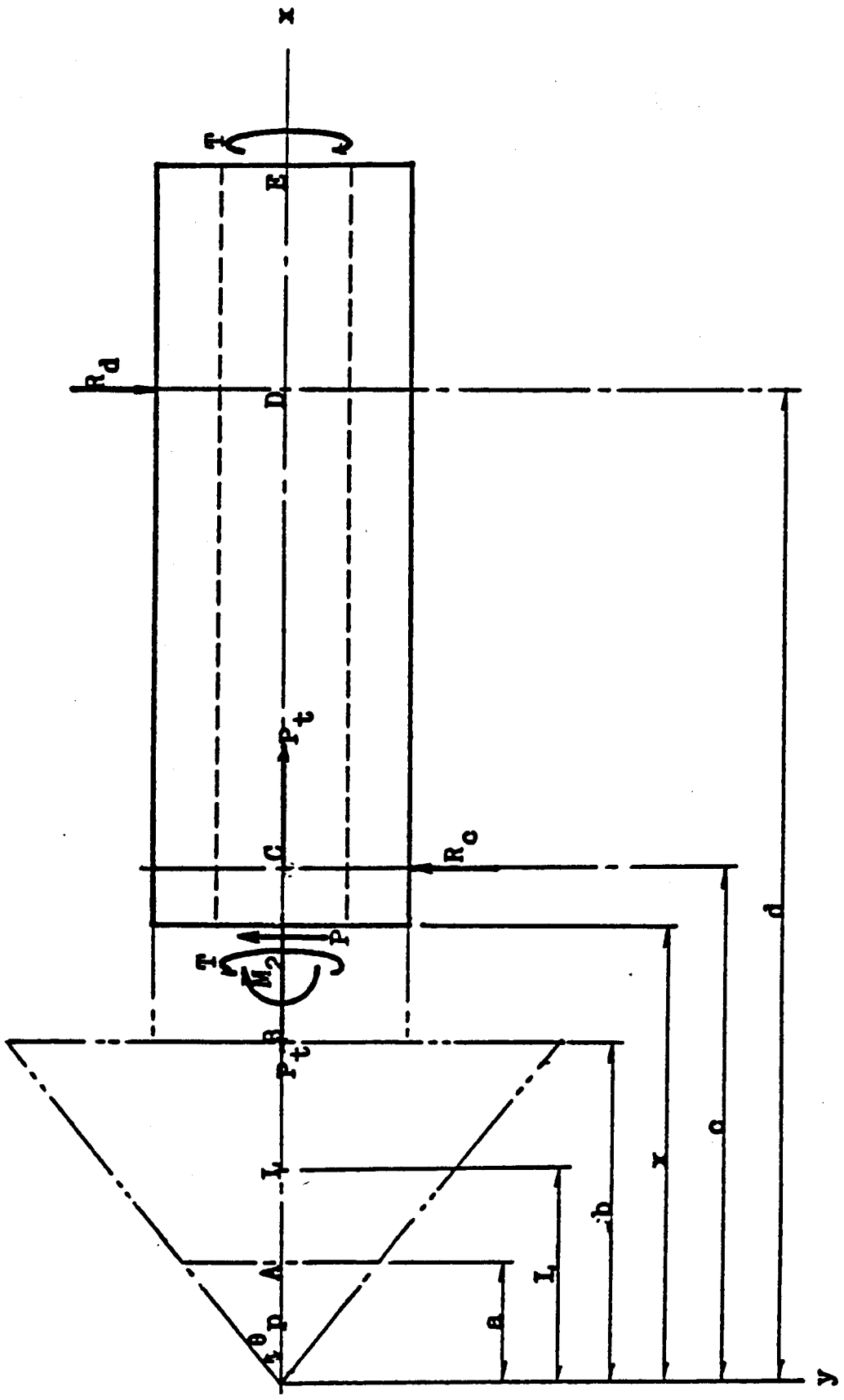

Fig. B-2 Free body diagram of the beam to the right of point B

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16. Abstract

A simplified analysis for obtaining the flexibility matrix of the contacting teeth of a pair of spiral bevel gears is presented. An existing finite element code SAP IV was used for computing the tooth deformation. The total deformation is obtained by superimposing the tooth deformation on the shaft deformation which was obtained from conventional beam theory. This simplified analysis was incorporated in the main computer code developed earlier by Chao (NASA CR-4077) for computing dynamic loads in spiral bevel gears.

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